$b \rightarrow s$ transitions in family-dependent $U(1)$ ' models

This article has been downloaded from IOPscience. Please scroll down to see the full text article.
JHEP12(2009)048
(http://iopscience.iop.org/1126-6708/2009/12/048)
The Table of Contents and more related content is available

Download details:
IP Address: 80.92.225.132
The article was downloaded on 01/04/2010 at 13:19

Please note that terms and conditions apply.

Received: August 25, 2009
Accepted: November 24, 2009
Published: December 15, 2009

## $b \rightarrow s$ transitions in family-dependent $\mathrm{U}(1)^{\prime}$ models

Large Vernon Barger, ${ }^{a}$ Lisa L. Everett, ${ }^{a}$ Jing Jiang, ${ }^{a}$ Paul Langacker, ${ }^{b}$ Tao Liu ${ }^{c}$ and Carlos E.M. Wagner ${ }^{c, d, e}$<br>${ }^{a}$ Department of Physics, Universityof Wisconsin, 1150 University Avenue, Madison, WI 53706, U.S.A.<br>${ }^{b}$ School of Natural Science, Institute for Advanced Study, Einstein Drive, NJ 08540, Princeton U.S.A.<br>${ }^{\text {c }}$ Enrico Fermi Institute, University of Chicago, 5640 S. Ellis Ave., IL 60637, Chicago U.S.A.<br>${ }^{d}$ Kavli Institute for Cosmological Physics, University of Chicago, 5640 S. Ellis Ave., IL 60637, Chicago U.S.A.<br>${ }^{e}$ HEP Division, Argonne National Laboratory, 9700 Cass Ave., IL 60439, Argonne U.S.A.<br>E-mail: barger@pheno.wisc.edu, leverett@wisc.edu, jiangj07@gmail.com, pgl@ias.edu, taoliu@theory.uchicago.edu, cwagner@hep.anl.gov

Abstract: We analyze flavor-changing-neutral-current (FCNC) effects in the $b \rightarrow s$ transitions that are induced by family non-universal $\mathrm{U}(1)^{\prime}$ gauge symmetries. After systematically developing the necessary formalism, we present a correlated analysis for the $\Delta B=1,2$ processes. We adopt a model-independent approach in which we only require familyuniversal charges for the first and second generations and small fermion mixing angles. We analyze the constraints on the resulting parameter space from $B_{s}-\bar{B}_{s}$ mixing and the time-dependent CP asymmetries of the penguin-dominated $B_{d} \rightarrow\left(\pi, \phi, \eta^{\prime}, \rho, \omega, f_{0}\right) K_{S}$ decays. Our results indicate that the currently observed discrepancies in some of these modes with respect to the Standard Model predictions can be consistently accommodated within this general class of models.

Keywords: Beyond Standard Model, B-Physics, CP violation

ArXiv ePrint: 0906.3745

## Contents

1 Introduction ..... 1
2 Theoretical background ..... 4
2.1 Formalism of $Z^{\prime}$-induced FCNC effects ..... 4
2.2 Effective couplings at the $b$ mass scale ..... 12
3 Results and analysis ..... 14
3.1 Correlated analysis (I). Special limits ..... 14
3.2 Correlated analysis (II). General case ..... 22
4 Conclusions ..... 25
A Operators of effective hamiltonians ..... 25
B Constraints of $\operatorname{Br}\left(\boldsymbol{B}_{\boldsymbol{s}} \rightarrow \boldsymbol{\mu}^{+} \boldsymbol{\mu}^{-}\right)$ ..... 26
C Parameters ..... 27

## 1 Introduction

The origin of $C P$ violation, which was first observed in the kaon system four decades ago [1], has remained one of the fundamental questions of elementary particle physics. In recent years, the B factories have established that the Standard Model (SM) picture of $C P$ violation, in which all $C P$-violating effects are generated by the single phase $\delta_{\mathrm{CKM}}$ in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix [2, 3], is consistent with the observed pattern of $C P$-violating phenomena in both the $B_{d}$ and $K$ meson systems [4]. However, as the SM cannot account for the baryon asymmetry in the Universe today [5], new physics (NP) is necessarily required to describe all observed phenomena with $C P$ violation involved.

One arena to seek the NP effects is in flavor-changing neutral current transitions (FCNC) where the SM contributions first appear at the one-loop level and the NP effects can be competitive. The emblematic set of such processes is the set of $b \rightarrow s$ transitions, which include $B_{s}-\bar{B}_{s}$ mixing and the set of neutral $B_{d}$ meson decays which occur via $b \rightarrow s \bar{q} q(q=u, d, c, s)$ transitions. Several of these processes are also of interest because recent measurements exhibit discrepancies with the SM predictions at the level of a few standard deviations, which may suggest the intriguing possibility of physics beyond the SM. The current status of the data is as follows:

| Observable | $1 \sigma$ C.L. | $2 \sigma$ C.L. |
| :---: | :---: | :---: |
| $\phi_{B_{s}}^{\mathrm{NP}}\left[{ }^{\circ}\right](\mathrm{S} 1)$ | $-20.3 \pm 5.3$ | $[-30.5,-9.9]$ |
| $\phi_{B_{s}}^{\mathrm{NP}}\left[{ }^{\circ}\right](\mathrm{S} 2)$ | $-68.0 \pm 4.8$ | $[-77.8,-58.2]$ |
| $C_{B_{s}}$ | $1.00 \pm 0.20$ | $[0.68,1.51]$ |
| $\phi_{s}^{\mathrm{NP}}\left[{ }^{\circ}\right](\mathrm{S} 1)$ | $-56.3 \pm 8.3$ | $[-69.8,-36.0]$ |
| $A_{s}^{\mathrm{NP}} / A_{s}^{\mathrm{SM}}(\mathrm{S} 1)$ | $0.66 \pm 0.28$ | $[0.24,1.11]$ |
| $\phi_{s}^{\mathrm{NP}}\left[{ }^{\circ}\right](\mathrm{S} 2)$ | $-79.1 \pm 2.6$ | $[-84.0,-72.8]$ |
| $A_{s}^{\mathrm{NP}} / A_{s}^{\mathrm{SM}}(\mathrm{S} 2)$ | $1.78 \pm 0.03$ | $[1.53,2.19]$ |

Table 1. Fit results for the $B_{s}-\bar{B}_{s}$ mixing parameters [6]. The two $\phi_{B_{s}}^{\mathrm{NP}}$ solutions ("S1" and "S2") result from measurement ambiguities; see [6] for details.

- $\boldsymbol{B}_{\boldsymbol{s}}-\overline{\boldsymbol{B}}_{\boldsymbol{s}}$ mixing phase. The standard way to parametrize NP in $B_{s}-\bar{B}_{s}$ mixing is to express the off-diagonal mixing matrix element as follows:

$$
\begin{equation*}
M_{12}^{B_{s}}=\left(M_{12}^{B_{s}}\right)_{\mathrm{SM}} C_{B_{s}} e^{2 i \phi_{B_{s}}^{\mathrm{NP}}} \tag{1.1}
\end{equation*}
$$

The SM predicts that $C_{B_{s}}=1$ and $\phi_{B_{s}}^{\mathrm{NP}}=0$. Though the data indicate that $C_{B_{s}}$ does not differ significantly from unity, the results of a recent analysis [6] suggest that $\phi_{B_{s}}^{\mathrm{NP}}$ deviates from zero at the $3 \sigma$ level (see table 1) (also see [7] for an earlier discussion on this discrepancy). This analysis combines all the available experimental results on $B_{s}$ mixing, including the new tagged analyses of $B_{s} \rightarrow \psi \phi$ by CDF [8] and D $\emptyset$ [9] (note that no single measurement yet has a $3 \sigma$ significance.). The discrepancy disfavors NP scenarios which obey minimal flavor violation (MFV), i.e., with $\phi_{B_{s}}^{\mathrm{NP}} \approx 0$, and instead suggests NP which exhibits flavor violation in the $b \rightarrow s$ transitions (e.g., see [10] and references therein). For convenience, in table 1 we also give the data in terms of $A_{s}^{\mathrm{NP}} / A_{s}^{\mathrm{SM}}$ and $\phi_{s}^{\mathrm{NP}}$ which are related to $C_{B_{s}}$ and $\phi_{B_{s}}^{\mathrm{NP}}$ according to

$$
\begin{equation*}
C_{B_{s}} e^{2 i \phi_{B_{s}}^{\mathrm{NP}}}=1+\frac{A_{s}^{\mathrm{NP}}}{A_{s}^{\mathrm{SM}}} e^{2 i \phi_{s}^{\mathrm{NP}}} \tag{1.2}
\end{equation*}
$$

- $\boldsymbol{C P}$ asymmetries in neutral $\boldsymbol{B}_{\boldsymbol{d}}$ decays. The set of neutral $B_{d}$ decays in question is the set of QCD penguin-dominated charmless decays that occur via $b \rightarrow s \bar{q} q$ $(q=u, d, c, s)$ transitions. The $C P$ asymmetries of such decays into a final $C P$ eigenstate $f_{C P}$ are given by

$$
\begin{align*}
\mathcal{A}_{f_{C P}}(t) & =\left.\frac{\Gamma\left(\bar{B}_{d}(t) \rightarrow f_{C P}\right)-\Gamma\left(B_{d}(t) \rightarrow f_{C P}\right)}{\Gamma\left(\bar{B}_{d}(t) \rightarrow f_{C P}\right)+\Gamma\left(B_{d}(t) \rightarrow f_{C P}\right)}\right|_{\Delta \Gamma_{B_{d}}=0} \\
& =-\mathcal{C}_{f_{C P}} \cos \left(\Delta M_{B_{d}} t\right)+\mathcal{S}_{f_{C P}} \sin \left(\Delta M_{B_{d}} t\right) \tag{1.3}
\end{align*}
$$

in which $\mathcal{C}_{f_{C P}}$ and $\mathcal{S}_{f_{C P}}$ are direct and mixing-induced $C P$ asymmetry parameters. The SM predictions for many decays of this type, including $B_{d} \rightarrow \psi K_{S}$ and $B_{d} \rightarrow$ $\left(\phi, \eta^{\prime}, \pi, \rho, \omega, f_{0}\right) K_{S}$ are as follows:

$$
\begin{equation*}
-\eta_{f_{C P}} \mathcal{S}_{f_{C P}}=\sin 2 \beta+\mathcal{O}\left(\lambda^{2}\right), \quad \mathcal{C}_{f_{C P}}=0+\mathcal{O}\left(\lambda^{2}\right) \tag{1.4}
\end{equation*}
$$

| $f_{C P}$ | $-\eta_{C P} \mathcal{S}_{f_{C P}}(1 \sigma$ C.L. $)$ | $\mathcal{C}_{f_{C P}}(1 \sigma$ C.L. $)$ |
| :---: | :---: | :---: |
| $\psi K_{S}$ | $+0.672 \pm 0.024$ | $+0.005 \pm 0.019$ |
| $\phi K_{S}$ | $+0.44_{-0.18}^{+0.17}$ | $-0.23 \pm 0.15$ |
| $\eta^{\prime} K_{S}$ | $+0.59 \pm 0.07$ | $-0.05 \pm 0.05$ |
| $\pi K_{S}$ | $+0.57 \pm 0.17$ | $+0.01 \pm 0.10$ |
| $\rho K_{S}$ | $+0.63_{-0.21}^{+0.17}$ | $-0.01 \pm 0.20$ |
| $\omega K_{S}$ | $+0.45 \pm 0.24$ | $-0.32 \pm 0.17$ |
| $f_{0} K_{S}$ | $+0.62_{-0.13}^{+0.11}$ | $0.10 \pm 0.13$ |

Table 2. World averages of the experimental results for the $C P$ asymmetries in $B_{d}$ decays via $b \rightarrow \bar{q} q s$ transitions [11].
with $\beta \equiv \arg \left[-\left(V_{c d} V_{c b}^{*}\right) /\left(V_{t d} V_{t b}^{*}\right)\right], \lambda=\sin \theta_{c}$ being the Cabibbo angle, and $\eta_{f_{C P}}= \pm 1$ being the $C P$ eigenvalue for the final state $f_{C P}$. However, the central values of $\sin 2 \beta$ directly measured from the penguin-dominated modes are systematically below the SM prediction and the results obtained from measuring the charmed $B_{d} \rightarrow \psi K_{S}$ mode. Meanwhile, the central values of the direct $C P$ asymmetry measured from $B_{d} \rightarrow \phi K_{S}$ and $B_{d} \rightarrow \omega K_{S}$ modes are also small compared to that obtained from the $B_{d} \rightarrow \psi K_{S}$ mode (see table 2). Given that the $B_{d} \rightarrow \psi K_{S}$ decay is dominated by treelevel amplitude in the SM , large absolute values for $\Delta \mathcal{S}_{f_{C P}}=-\eta_{f_{C P}} \mathcal{S}_{f_{C P}}+\eta_{\psi K_{S}} \mathcal{S}_{\psi K_{S}}$ and $\Delta \mathcal{C}_{f_{C P}}=\mathcal{C}_{f_{C P}}-\mathcal{C}_{\psi K_{S}}$ may imply interesting NP in the $b \rightarrow s$ transitions.

To account for these discrepancies appearing in $B_{s}-\bar{B}_{s}$ mixing and $B_{d}$ decays, a number of NP scenarios have been studied [12]. In many of these scenarios, the effects of NP in the $b \rightarrow s$ transitions are loop-suppressed and can compete with SM contributions. The most popular and well-studied scenarios are models with MFV, in which the only source of $C P$ violation is the single irremovable phase of the CKM mixing matrix. MFV scenarios, however, face difficulties in that they do not generally allow for a nonvanishing $\phi_{B_{s}}$.

In this work, we will study the constraints from $b \rightarrow s$ transitions on models with family non-universal gauged $U(1)^{\prime}$ symmetries. Family non-universal $U(1)^{\prime}$ gauge symmetries are present in many well-motivated extensions of the SM. For example, they are ubiquitous in semirealistic string compactifications, both within perturbative heterotic models (e.g., the free fermionic models $[13,14]$ ) and Type II string models (e.g., the intersecting brane models [15]). Family non-universal $\mathrm{U}(1)^{\prime}$ symmetries are also easily obtained within extra-dimensional models, such as warped Randall-Sundrum models that address the flavor problem where the relevant $Z^{\prime}$ gauge boson can be either the zero or the excited KaluzaKlein modes (for recent discussion, see [16]). Ultimately, the underlying reason is that it is typical for the third family to have different properties than the other two families, often for fermion mass generation. Such scenarios are of particular interest because unlike the scenarios studied above, they allow for the intriguing possibility of tree-level FCNC, with contributions that are competitive with the SM even for small $\mathrm{U}(1)^{\prime}$ couplings. Depending on the details of the model, family-dependent $U(1)^{\prime}$ scenarios can result in new FC operators and/or modified Wilson coefficients to the existing SM operators in the oper-
ator product expansion，providing a rich framework beyond MFV to explore FCNC and $C P$－violating effects．

We follow the general framework for addressing $Z^{\prime}$－induced FCNC as developed in［18］ and systemize its application to $b \rightarrow s$ transitions．Rather than considering specific $\mathrm{U}(1)^{\prime}$ models mentioned above，we adopt a model－independent approach in which the main re－ strictions are family universal charges for the first and second generations and small fermion mixing angles．The formalism developed in subsection 2.1 and the analyses in the subse－ quent sections，therefore，could be applied to any models in which there exist tree－level FCNC effects induced by such heavy neutral gauge bosons（this heavy neutral gauge boson can even be from non－Abelian gauge symmetry）．In this work，we also neglect the effects of $Z-Z^{\prime}$ mixing（which are known to be small），and assume the absence of any exotic fermions that could mix with the usual SM fermions through non－universal $Z^{\prime}$ couplings， which may also result in nontrivial FCNC effects（e．g．，see［17］）．

This work is an extension of our earlier work［19］，in which we performed a correlated analysis of the $\Delta B=1,2$ processes mentioned above for a specific set of $\mathrm{U}(1)^{\prime}$ scenarios． That analysis was in contrast to other studies of $\mathrm{U}(1)^{\prime}$ scenarios based on mode－by－mode analyses［20，21］．The purpose of this paper is twofold：first，to provide more details of the formalism and analysis than were given explicitly in［19］，and second，to analyze a more general set of $\mathrm{U}(1)^{\prime}$ models．Our results demonstrate that the $b \rightarrow s$ transitions not only place important constraints on family non－universal $Z^{\prime}$ couplings and mass scale，but also that family non－universal $\mathrm{U}(1)^{\prime}$ scenarios can explain the currently observed discrepancies with the SM predictions for $B_{s}-\bar{B}_{s}$ mixing and the time－dependent $C P$ asymmetries of the penguin－dominated $B_{d} \rightarrow\left(\pi, \phi, \eta^{\prime}, \rho, \omega, f_{0}\right) K_{S}$ decays．

This paper is structured as follows．We begin by providing an overview of the for－ malism of the $Z^{\prime}$ induced FCNC effects in the $b \rightarrow s$ transitions and present the effective Hamiltonian for the processes of interest at the $b$ quark mass scale in section 2．In section 3 ，first we analyze the FCNC constraints within several special limits of the general $\mathrm{U}(1)^{\prime}$ parameter space，and then turn to a more general analysis．Our summary and conclusions are presented in section 4.

## 2 Theoretical background

## 2．1 Formalism of $Z^{\prime}$－induced FCNC effects

The general framework for studying $Z^{\prime}$－induced FCNC effects has been developed in［18］． In this section，we will systematically formalize its applications to the case of the $b \rightarrow s$ transitions（the generalization to $b \rightarrow d$ transitions is straightforward）．

We begin by considering the SM extended by a single additional $\mathrm{U}(1)^{\prime}$ gauge symmetry （the generalization to multiple $\mathrm{U}(1)^{\prime}$ gauge symmetries is straightforward）．In this theory， the neutral current Lagrangian in the SM gauge eigenstate basis is given by

$$
\begin{equation*}
\mathcal{L}_{N C}=-e J_{\mathrm{em}}^{\mu} A_{\mu}-g_{1} J_{Z}^{\mu} Z_{\mu}-g_{2} J_{Z}^{\mu} Z_{\mu}^{\prime}, \tag{2.1}
\end{equation*}
$$

in which $A_{\mu}$ is the $\mathrm{U}(1)_{\text {em }}$ gauge boson，$Z_{\mu}$ is the massive electroweak（EW）neutral gauge boson，$Z_{\mu}^{\prime}$ is the gauge boson associated with the additional Abelian gauge symmetry，and
$g_{1}=g / \cos \theta_{W}$ and $g_{2}$ are the gauge couplings of the $Z_{\mu}$ and $Z_{\mu}^{\prime}$ bosons, respectively. The currents are given by

$$
\begin{align*}
J_{Z}^{\mu} & =\sum_{\psi} \sum_{i} \bar{\psi}_{i} \gamma^{\mu}\left[\epsilon_{i}^{\psi_{L}} P_{L}+\epsilon_{i}^{\psi_{R}} P_{R}\right] \psi_{i}  \tag{2.2}\\
J_{Z^{\prime}}^{\mu} & =\sum_{\psi} \sum_{i, j} \bar{\psi}_{i} \gamma^{\mu}\left[\tilde{\epsilon}_{i j}^{\psi_{L}} P_{L}+\tilde{\epsilon}_{i j}^{\psi_{R}} P_{R}\right] \psi_{j} \tag{2.3}
\end{align*}
$$

in which $\psi$ labels the SM fermions, $i$ and $j$ are family indices, and $P_{R, L}=\left(1 \pm \gamma_{5}\right) / 2$. The (family universal) SM chiral charges are given by

$$
\begin{equation*}
\epsilon_{i}^{\psi_{L}}=t_{3}^{\psi_{L}}-\sin ^{2} \theta_{W} Q_{\psi_{L}}, \quad \epsilon_{i}^{\psi_{R}}=-\sin ^{2} \theta_{W} Q_{\psi_{R}} \tag{2.4}
\end{equation*}
$$

in which $t_{3}^{\psi_{L}}$ denotes the third component of the weak isospin and $Q_{\psi_{L, R}}$ are the electric charges of $\psi_{L, R}$. Without loss of generality, the $Z^{\prime}$ chiral charges can be diagonalized by choosing the appropriate gauge basis for the fermions:

$$
\begin{equation*}
\tilde{\epsilon}_{i j}^{\psi_{L, R}}=\tilde{\epsilon}_{i}^{\psi_{L, R}} \delta_{i j} . \tag{2.5}
\end{equation*}
$$

In particular, $\mathrm{SU}(2)_{L}$ symmetry requires that ${ }^{1}$

$$
\begin{equation*}
\tilde{\epsilon}_{i}^{u_{L}} \equiv \tilde{\epsilon}_{i}^{d_{L}}, \quad \tilde{\epsilon}_{i}^{e_{L}} \equiv \tilde{\epsilon}_{i}^{\nu_{L}} \tag{2.6}
\end{equation*}
$$

If the diagonal $\mathrm{U}(1)^{\prime}$ chiral charges are non-universal, flavor-changing (FC) $Z^{\prime}$ couplings are generically induced by fermion mixing. The fermion Yukawa matrices $h_{\psi}$ in the weak eigenstate basis are diagonalized by the unitary matrices $V_{\psi_{L, R}}$, such that

$$
\begin{equation*}
h_{\psi, \text { diag }}=V_{\psi_{R}} h_{\psi} V_{\psi_{L}}^{\dagger} \tag{2.7}
\end{equation*}
$$

and the CKM matrix is given by

$$
\begin{equation*}
V_{\mathrm{CKM}}=V_{u_{L}} V_{d_{L}}^{\dagger} \tag{2.8}
\end{equation*}
$$

Hence, the chiral $Z^{\prime}$ couplings in the fermion mass eigenstate basis take the form:

$$
\begin{equation*}
B^{\psi_{L}} \equiv V_{\psi_{L}} \tilde{\epsilon}^{\psi_{L}} V_{\psi_{L}}^{\dagger}, \quad B^{\psi_{R}} \equiv V_{\psi_{R}} \tilde{\epsilon}^{\psi_{R}} V_{\psi_{R}}^{\dagger} \tag{2.9}
\end{equation*}
$$

However, it is known that the constraints from $K-\bar{K}$ mixing and from $\mu-e$ conversion in muonic atoms exclude significant non-universal effects for the first two families, which suggests that

$$
B^{\psi_{L, R}}=\left(\begin{array}{ccc}
B_{11}^{\psi_{L, R}} & 0 & B_{13}^{\psi_{L, R}}  \tag{2.10}\\
0 & B_{11}^{\psi_{L, R}} & B_{23}^{\psi_{L, R}} \\
B_{13}^{\psi_{L, R} *} & B_{23}^{\psi_{L, R} *} & B_{33}^{\psi_{L, R}}
\end{array}\right)
$$

[^0]at least for the down-type quarks and $e, \mu, \tau$ leptons. The most straightforward way to achieve this coupling structure is to assume universal $\mathrm{U}(1)^{\prime}$ charges for the down-type fermions of the first two families, i.e.,
\[

\tilde{\epsilon}^{\psi_{L, R}}=\left($$
\begin{array}{ccc}
\tilde{\epsilon}_{1}^{\psi_{L, R}} & 0 & 0  \tag{2.11}\\
0 & \tilde{\epsilon}_{1}^{\psi_{L, R}} & 0 \\
0 & 0 & \tilde{\epsilon}_{3}^{\psi_{L, R}}
\end{array}
$$\right)
\]

With the unitary matrices $V_{\psi_{L, R}}$ written as

$$
V_{\psi_{L, R}}=\left(\begin{array}{cc}
W_{\psi_{L, R}} & X_{\psi_{L, R}}  \tag{2.12}\\
Y_{\psi_{L, R}} & Z_{\psi_{L, R}}
\end{array}\right)
$$

where $W_{\psi_{L, R}}$ is a $2 \times 2$ submatrix, one obtains

$$
\begin{equation*}
B^{\psi_{L, R}}=\binom{\tilde{\epsilon}_{1}^{\psi_{L, R}} W_{\psi_{L, R}}^{\dagger} W_{\psi_{L, R}}+\tilde{\epsilon}_{3}^{\psi_{L, R}} Y_{\psi_{L, R}}^{\dagger} Y_{\psi_{L, R}} \tilde{\epsilon}_{1}^{\psi_{L, R}} W_{\psi_{L, R}}^{\dagger} X_{\psi_{L, R}}+\tilde{\epsilon}_{3}^{\psi_{L, R}} Y_{\psi_{L, R}}^{\dagger} Z_{\psi_{L, R}}}{\tilde{\epsilon}_{1}^{\psi_{L, R}} X_{\psi_{L, R}}^{\dagger} W_{\psi_{L, R}}+\tilde{\epsilon}_{3}^{\psi_{L, R}} Z_{\psi_{L, R}}^{\dagger} Y_{\psi_{L, R}} \tilde{\epsilon}_{1}^{\psi_{L, R}} X_{\psi_{L, R}}^{\dagger} X_{\psi_{L, R}}+\tilde{\epsilon}_{3}^{\psi_{L, R}} Z_{\psi_{L, R}}^{\dagger} Z_{\psi_{L, R}}} . \tag{2.13}
\end{equation*}
$$

Therefore, in the limit of small fermion mixing angles or small $X_{\psi_{L, R}}, Y_{\psi_{L, R}}$ elements, a $Z^{\prime}$ coupling structure of the type given in eq. (2.10) is produced, in which

$$
\begin{align*}
& B_{11}^{\psi_{L, R}}=\tilde{\epsilon}_{1}^{\psi_{L, R}}, \quad B_{33}^{\psi_{L, R}}=\tilde{\epsilon}_{3}^{\psi_{L, R}} \\
& B_{13}^{\psi_{L, R}}, B_{23}^{\psi_{L, R}} \sim \mathcal{O}\left(X_{\psi_{L, R}}, Y_{\psi_{L, R}}\right), \tag{2.14}
\end{align*}
$$

such that $B_{13}^{\psi_{L, R}}$ and $B_{23}^{\psi_{L, R}}$ are in general both complex parameters.
EW symmetry breaking induces $Z-Z^{\prime}$ mixing, such that the gauge eigenstates $Z_{\mu}$ and $Z_{\mu}^{\prime}$ are related to the mass eigenstates $Z_{\mu}^{(n)}(n=1,2)$ by an orthogonal transformation. In the mass eigenstate basis, the Lagrangian couplings are given by

$$
\begin{equation*}
\mathcal{L}_{N C}^{Z}=-\left[g_{1} \cos \theta J_{Z}^{\mu}+g_{2} \sin \theta J_{Z^{\prime}}^{\mu}\right] Z_{\mu}^{(1)}-\left[-g_{1} \sin \theta J_{Z}^{\mu}+g_{2} \cos \theta J_{Z^{\prime}}^{\mu}\right] Z_{\mu}^{(2)}, \tag{2.15}
\end{equation*}
$$

where $\theta$ is the $Z-Z^{\prime}$ mixing angle, $J_{Z}^{\mu}$ is given in eq. (2.2), and $J_{Z^{\prime}}^{\mu}$ is of the form of eq. (2.3) with $\tilde{\epsilon}^{\psi_{L, R}}$ replaced by $B^{\psi_{L, R}}$ from eq. (2.9). In this analysis, we neglect kinetic mixing since it simply amounts to a redefinition of the unknown $Z^{\prime}$ couplings. ${ }^{2}$

At the EW scale, the tree-level four-fermion interactions are described by the product of gauge currents

$$
\begin{align*}
\mathcal{L}_{\mathrm{eff}} & =\frac{-4 G_{F}}{\sqrt{2}}\left(\rho_{\mathrm{eff}} J_{Z}{ }^{2}+2 w J_{Z} \cdot J_{Z^{\prime}}+y J_{Z^{\prime}}{ }^{2}\right) \\
& =\frac{-4 G_{F}}{\sqrt{2}} \sum_{\psi, \chi} \sum_{i, j, m, n}\left[C_{m n}^{i j} S_{m n}^{i j}+\tilde{C}_{m n}^{i j} \tilde{S}_{m n}^{i j}+D_{m n}^{i j} T_{m n}^{i j}+\tilde{D}_{m n}^{i j} \tilde{T}_{m n}^{i j}\right] \tag{2.16}
\end{align*}
$$

[^1]In eq. (2.16), the local current-current operators are ${ }^{3}(i, j, m, n$ are family indices):

$$
\begin{array}{ll}
S_{m n}^{i j}=\left(\bar{\psi}_{i} \gamma^{\mu} P_{L} \psi_{j}\right)\left(\bar{\chi}_{m} \gamma_{\mu} P_{L} \chi_{n}\right), & \tilde{S}_{m n}^{i j}=\left(\bar{\psi}_{i} \gamma^{\mu} P_{R} \psi_{j}\right)\left(\bar{\chi}_{m} \gamma_{\mu} P_{R} \chi_{n}\right), \\
T_{m n}^{i j}=\left(\bar{\psi}_{i} \gamma^{\mu} P_{L} \psi_{j}\right)\left(\bar{\chi}_{m} \gamma_{\mu} P_{R} \chi_{n}\right), & \tilde{T}_{m n}^{i j}=\left(\bar{\psi}_{i} \gamma^{\mu} P_{R} \psi_{j}\right)\left(\bar{\chi}_{m} \gamma_{\mu} P_{L} \chi_{n}\right), \tag{2.17}
\end{array}
$$

and the coefficients are

$$
\begin{align*}
& C_{m n}^{i j}=\rho_{e f f} \delta_{i j} \delta_{m n} \epsilon_{i}^{\psi_{L}} \epsilon_{m}^{\chi_{L}}+w \delta_{i j} \epsilon_{i}^{\psi_{L}} B_{m n}^{\chi_{L}}+w \delta_{m n} \epsilon_{m}^{\chi_{L}} B_{i j}^{\psi_{L}}+y B_{i j}^{\psi_{L}} B_{m n}^{\chi_{L}}, \\
& \tilde{C}_{m n}^{i j}=\rho_{e f f} \delta_{i j} \delta_{m n} \epsilon_{i}^{\psi_{R}} \epsilon_{m}^{\chi_{R}}+w \delta_{i j} \epsilon_{i}^{\psi_{R}} B_{m n}^{\chi_{R}}+w \delta_{m n} \epsilon_{m}^{\chi_{R}} B_{i j}^{\psi_{R}}+y B_{i j}^{\psi_{R}} B_{m n}^{\chi_{R}}, \\
& D_{m n}^{i j}=\rho_{e f f} \delta_{i j} \delta_{m n} \epsilon_{i}^{\psi_{L} \epsilon_{m}^{\chi_{R}}+w \delta_{i j} \epsilon_{i}^{\psi_{L}} B_{m n}^{\chi_{R}}+w \delta_{m n} \epsilon_{m}^{\chi_{R}} B_{i j}^{\psi_{L}}+y B_{i j}^{\psi_{L}} B_{m n}^{\chi_{R}},} \\
& \tilde{D}_{m n}^{i j}=\rho_{e f f} \delta_{i j} \delta_{m n} \epsilon_{i}^{\psi_{R}} \epsilon_{m}^{\chi_{L}}+w \delta_{i j} \epsilon_{i}^{\psi_{R}} B_{m n}^{\chi_{L}}+w \delta_{m n} \epsilon_{m}^{\chi_{L}} B_{i j}^{\psi_{R}}+y B_{i j}^{\psi_{R}} B_{m n}^{\chi_{L}}, \tag{2.18}
\end{align*}
$$

in which

$$
\begin{align*}
\rho_{e f f} & =\rho_{1} \cos ^{2} \theta+\rho_{2} \sin ^{2} \theta, \quad \rho_{a}=\frac{M_{W}^{2}}{M_{a}^{2} \cos ^{2} \theta_{W}}, \\
w & =\frac{g_{2}}{g_{1}} \sin \theta \cos \theta\left(\rho_{1}-\rho_{2}\right), \\
y & =\left(\frac{g_{2}}{g_{1}}\right)^{2}\left(\rho_{1} \sin ^{2} \theta+\rho_{2} \cos ^{2} \theta\right) . \tag{2.19}
\end{align*}
$$

In eqs. (2.19), $M_{a}$ denotes the masses of the neutral gauge boson mass eigenstates, and $\theta_{W}$ is the EW mixing angle. We do not specify the $\psi$ and $\chi$ dependence of the coefficients $C, \tilde{C}, D, \tilde{D}$ in eqs. (2.18), which can be understood from the context.

For $b \rightarrow s$ transitions, the local operators are given by $S_{m n}^{b s}, \tilde{S}_{m n}^{b s}, T_{m n}^{b s}$ and $\tilde{T}_{m n}^{b s}$, with coefficients that are given by

$$
\begin{align*}
& C_{m n}^{b s}=w \delta_{m n} \epsilon_{m}^{\chi_{L}} B_{b s}^{L}+y B_{b s}^{L} B_{m n}^{\chi_{L}}, \\
& \tilde{C}_{m n}^{b s}=w \delta_{m n} \epsilon_{m}^{\chi_{R}} B_{b s}^{R}+y B_{b s}^{R} B_{m n}^{\chi_{R}}, \\
& D_{m n}^{b s}=w \delta_{m n} \epsilon_{m}^{\chi_{R}} B_{b s}^{L}+y B_{b s}^{L} B_{m n}^{\chi_{R}}, \\
& \tilde{D}_{m n}^{b s}=w \delta_{m n} \epsilon_{m}^{\chi_{R}} B_{b s}^{R}+y B_{b s}^{R} B_{m n}^{\chi_{L}} . \tag{2.20}
\end{align*}
$$

With the $Z-Z^{\prime}$ mixing angle neglected, the coefficients can be written as

$$
\begin{align*}
C_{m n}^{b s} & =\left(\frac{g_{2} M_{Z}}{g_{1} M_{Z^{\prime}}}\right)^{2} B_{b s}^{L} B_{m n}^{\chi_{L}}, \\
\tilde{C}_{m n}^{b s} & =\left(\frac{g_{2} M_{Z}}{g_{1} M_{Z^{\prime}}}\right)^{2} B_{b s}^{R} B_{m n}^{\chi_{R}}, \\
D_{m n}^{b s} & =\left(\frac{g_{2} M_{Z}}{g_{1} M_{Z^{\prime}}}\right)^{2} B_{b s}^{L} B_{m n}^{\chi_{R}}, \\
\tilde{D}_{m n}^{b s} & =\left(\frac{g_{2} M_{Z}}{g_{1} M_{Z^{\prime}}}\right)^{2} B_{b s}^{R} B_{m n}^{\chi_{L}} . \tag{2.21}
\end{align*}
$$

[^2]For convenience, in the following we will resolve the factor $g_{2} M_{Z} /\left(g_{1} M_{Z^{\prime}}\right)$ into the $B$ elements or the chiral couplings.

At tree level, there are three classes of $b \rightarrow s$ transitions which are sensitive to the possible NP effects that result from an additional family non-universal $\mathrm{U}(1)^{\prime}$ symmetry: $b \rightarrow s \bar{q} q$ transitions, $b \rightarrow s \bar{l} l$ transitions, and $B_{s}-\bar{B}_{s}$ mixing. Here " $q$ " and "l" denote quarks and leptons, respectively. For the $b \rightarrow s \bar{q} q$ transitions, the $Z^{\prime}$ effects are described by the effective Hamiltonian

$$
\begin{align*}
\mathcal{H}_{\mathrm{eff}}^{Z^{\prime}}(b \rightarrow s \bar{q} q)=\frac{2 G_{F}}{\sqrt{2}}( & (\bar{s} b)_{V-A} \sum_{q}\left(C_{q q}^{b s}(\bar{q} q)_{V-A}+D_{q q}^{b s}(\bar{q} q)_{V+A}\right) \\
& \left.+(\bar{s} b)_{V+A} \sum_{q}\left(\tilde{D}_{q q}^{b s}(\bar{q} q)_{V-A}+\tilde{C}_{q q}^{b s}(\bar{q} q)_{V+A}\right)\right)+ \text { h.c. } \tag{2.22}
\end{align*}
$$

in which the sum is over the active quarks for a given process. These $Z^{\prime}$-induced FCNC effects can be understood as corrections to the SM operators or to the new penguin operators defined in A, since both lead to the same hadronic matrix elements. Explicitly, comparing eq. (2.22) with

$$
\begin{align*}
& \mathcal{H}_{\mathrm{eff}}^{Z^{\prime}}(b \rightarrow s \bar{q} q)  \tag{2.23}\\
& \quad-\frac{G_{F}}{\sqrt{2}} V_{t b} V_{t s}^{*}\left[(\bar{s} b)_{V-A} \sum_{q}\left(\left(\Delta C_{3}+\Delta C_{9} \frac{3}{2} e_{q}\right)(\bar{q} q)_{V-A}+\left(\Delta C_{5}+\Delta C_{7} \frac{3}{2} e_{q}\right)(\bar{q} q)_{V+A}\right)\right. \\
& \left.\quad+(\bar{s} b)_{V+A} \sum_{q}\left(\left(\Delta \tilde{C}_{3}+\Delta \tilde{C}_{9} \frac{3}{2} e_{q}\right)(\bar{q} q)_{V+A}+\left(\Delta \tilde{C}_{5}+\Delta \tilde{C}_{7} \frac{3}{2} e_{q}\right)(\bar{q} q)_{V-A}\right)\right]+ \text { h.c. }
\end{align*}
$$

results in $4 n_{q}$ equations ( $n_{q}$ is the number of active quarks in the final states):

$$
\begin{align*}
\Delta C_{3}+\Delta C_{9} \frac{3}{2} e_{q} & =\frac{-2}{V_{t b} V_{t s}^{*}} C_{q q}^{b s}, \\
\Delta C_{5}+\Delta C_{7} \frac{3}{2} e_{q} & =\frac{-2}{V_{t b} V_{t s}^{*}} D_{q q}^{b s}, \\
\Delta \tilde{C}_{3}+\Delta \tilde{C}_{9} \frac{3}{2} e_{q} & =\frac{-2}{V_{t b} V_{t s}^{*}} \tilde{C}_{q q}^{b s}, \\
\Delta \tilde{C}_{5}+\Delta \tilde{C}_{7} \frac{3}{2} e_{q} & =\frac{-2}{V_{t b} V_{t s}^{*}} \tilde{D}_{q q}^{b s}, \tag{2.24}
\end{align*}
$$

where $\Delta C$ denotes $Z^{\prime}$ correction to the Wilson coefficients of the SM operators and $\Delta \tilde{C}$ denotes the Wilson coefficients of the operators beyond the SM ones. For charmless processes with $q$ from the first two families, these equations are solvable because of the following relation obeyed by the down-type quark couplings:

$$
\begin{equation*}
B_{11}^{\psi_{L, R}}=B_{22}^{\psi_{L, R}} \tag{2.25}
\end{equation*}
$$

which is extracted from eq. (2.10). The $Z^{\prime}$ corrections to the Wilson coefficients are then found to be ${ }^{4}$

$$
\Delta C_{3}=-\frac{2}{3 V_{t b} V_{t s}^{*}}\left(C_{u u}^{b s}+2 C_{d d}^{b s}\right), \quad \Delta C_{9}=-\frac{4}{3 V_{t b} V_{t s}^{*}}\left(C_{u u}^{b s}-C_{d d}^{b s}\right)
$$

[^3]\[

$$
\begin{array}{rlr}
\Delta C_{5}=-\frac{2}{3 V_{t b} V_{t s}^{*}}\left(D_{u u}^{b s}+2 D_{d d}^{b s}\right), & \Delta C_{7}=-\frac{4}{3 V_{t b} V_{t s}^{*}}\left(D_{u u}^{b s}-D_{d d}^{b s}\right) \\
\Delta \tilde{C}_{3}=-\frac{2}{3 V_{t b} V_{t s}^{*}}\left(\tilde{C}_{u u}^{b s}+2 \tilde{C}_{d d}^{b s}\right), & \Delta \tilde{C}_{9}=-\frac{4}{3 V_{t b} V_{t s}^{*}}\left(\tilde{C}_{u u}^{b s}-\tilde{C}_{d d}^{b s}\right) \\
\Delta \tilde{C}_{5}=-\frac{2}{3 V_{t b} V_{t s}^{*}}\left(\tilde{D}_{u u}^{b s}+2 \tilde{D}_{d d}^{b s}\right), & \Delta \tilde{C}_{7}=-\frac{4}{3 V_{t b} V_{t s}^{*}}\left(\tilde{D}_{u u}^{b s}-\tilde{D}_{d d}^{b s}\right) . \tag{2.26}
\end{array}
$$
\]

We pause here to comment on subtleties in eq. (2.26). Recall that in the limit of small fermion mixing angles, eq. (2.14) holds for the down-type quarks. To obtain the CKM matrix as given in eq. (2.8) without requiring fine-tuned cancellations, the mixing angles for the up-type left-chiral quarks should also be small in this limit. Due to the $\mathrm{SU}(2)_{L}$ constraint of eq. (2.6), therefore, eq. (2.14) can also be applied to the up-type left-chiral quarks. In this case, it is straightforward to see that

$$
\begin{equation*}
B_{u u}^{L}-B_{d d}^{L} \approx \tilde{\epsilon}_{u u}^{L}-\tilde{\epsilon}_{d d}^{L} \equiv 0 \tag{2.27}
\end{equation*}
$$

and hence

$$
\begin{array}{ll}
\Delta C_{9} \approx 0, & \Delta \tilde{C}_{7} \approx 0 \\
\Delta C_{3} \approx-\frac{2}{V_{t b} V_{t s}^{*}} C_{d d}^{b s}, & \Delta \tilde{C}_{5} \approx-\frac{2}{V_{t b} V_{t s}^{*}} \tilde{D}_{d d}^{b s} \tag{2.28}
\end{array}
$$

Note that a relation similar to eq. (2.6) does not exist for the right-chiral SM fermions, so $\Delta C_{7}$ and $\Delta \tilde{C}_{9}$ are generically non-trivial. In regards to the color-allowed penguin operators, their Wilson coefficients are corrected by $Z^{\prime}$ effects only at the loop level where the colorindices are mixed by gluons. Since these effects suffer loop and $Z^{\prime}$ mass double suppressions, we will not consider them further in this paper.

For the $b \rightarrow s \bar{l} l$ transitions, the $Z^{\prime}$ contributions to the effective Hamiltonian are

$$
\begin{align*}
\mathcal{H}_{\mathrm{eff}}^{Z^{\prime}}(b \rightarrow s \bar{l} l)=\frac{2 G_{F}}{\sqrt{2}} & \left((\bar{s} b)_{V-A}\left(C_{l l}^{b s}(\bar{l})_{V-A}+D_{l l}^{b s}(\bar{l})_{V+A}\right)\right. \\
& +(\bar{s} b)_{V+A}\left(\tilde{D}_{l l}^{b s}(\overline{l l})_{V-A}+\tilde{C}_{l l}^{b s}(\bar{l} l)_{V+A}\right)+\text { h.c. } \tag{2.29}
\end{align*}
$$

Comparing eq. (2.29) with

$$
\begin{align*}
\mathcal{H}_{e f f}^{Z^{\prime}}(b \rightarrow s \bar{l} l)=-\frac{G_{F}}{\sqrt{2}} V_{t b} V_{t s}^{*}( & \Delta C_{9 V} Q_{9 V}+\Delta C_{10 A} Q_{10 A} \\
& \left.+\Delta \tilde{C}_{9 V} \tilde{Q}_{9 V}+\Delta \tilde{C}_{10 A} \tilde{Q}_{10 A}\right)+ \text { h.c. } \tag{2.30}
\end{align*}
$$

one can see that the $Z^{\prime}$ corrections to the Wilson coefficients take the following form:

$$
\Delta C_{9 V}=-\frac{2}{V_{t b} V_{t s}^{*}}\left(C_{l l}^{b s}+D_{l l}^{b s}\right)
$$

since it is only sensitive to the linear combinations on the left hand side of eq. (2.24). For the charmed processes where generally we have $n_{q}=1$, the formula in eq. (2.26) can also be applied as long as eq. (2.25) holds for the up-type quarks. If eq. (2.25) does not hold, then the " $u u$ " indices in these formula need to be replaced by " $c c$ ".

$$
\begin{align*}
\Delta C_{10 A} & =-\frac{2}{V_{t b} V_{t s}^{*}}\left(-C_{l l}^{b s}+D_{l l}^{b s}\right), \\
\Delta \tilde{C}_{9 V} & =-\frac{2}{V_{t b} V_{t s}^{*}}\left(\tilde{C}_{l l}^{b s}+\tilde{D}_{l l}^{b s}\right), \\
\Delta \tilde{C}_{10 A} & =-\frac{2}{V_{t b} V_{t s}^{*}}\left(\tilde{C}_{l l}^{b s}-\tilde{D}_{l l}^{b s}\right) . \tag{2.31}
\end{align*}
$$

Note that if the leptons in the process are neutrinos, eqs. (2.31) reduces to

$$
\begin{align*}
\Delta C_{9 V} & =-\frac{2}{V_{t b} V_{t s}^{*}} C_{l l}^{b s}, \\
\Delta C_{10 A} & =\frac{2}{V_{t b} V_{t s}^{*}} C_{l l}^{b s}, \\
\Delta \tilde{C}_{9 V} & =-\frac{2}{V_{t b} V_{t s}^{*}} \tilde{D}_{l l}^{b s}, \\
\Delta \tilde{C}_{10 A} & =\frac{2}{V_{t b} V_{t s}^{*}} \tilde{D}_{l l}^{b s}, \tag{2.32}
\end{align*}
$$

since right-handed neutrinos are generally decoupled at low energy scales.
For $B_{s}-\bar{B}_{s}$ mixing, the $Z^{\prime}$ corrections to the effective Hamiltonian take the form

$$
\begin{align*}
& \mathcal{H}_{\mathrm{eff}}^{Z^{\prime}}\left(B_{s}-\bar{B}_{s}\right)=\frac{G_{F}}{\sqrt{2}}\left(C_{b s}^{b s}(\bar{s} b)_{V-A}(\bar{s} b)_{V-A}+D_{b s}^{b s}(\bar{s} b)_{V-A}(\bar{s} b)_{V+A}\right. \\
&\left.+\tilde{D}_{b s}^{b s}(\bar{s} b)_{V+A}(\bar{s} b)_{V-A}+\tilde{C}_{b s}^{b s}(\bar{s} b)_{V+A}(\bar{s} b)_{V+A}\right)+ \text { h.c. } \tag{2.33}
\end{align*}
$$

Once again, upon comparing this expression to

$$
\begin{equation*}
\mathcal{H}_{e f f}^{Z^{\prime}}\left(B_{s}-\bar{B}_{s}\right)=-\frac{G_{F}}{\sqrt{2}}\left(\Delta C_{1}^{B_{s}} Q_{1}^{B_{s}}+\Delta \tilde{C}_{1}^{B_{s}} \tilde{Q}_{1}^{B_{s}}+2 \Delta \tilde{C}_{3}^{B_{s}} \tilde{Q}_{3}^{B_{s}}\right)+\text { h.c. } \tag{2.34}
\end{equation*}
$$

the $Z^{\prime}$ corrections to the Wilson coefficients are easily determined to be

$$
\begin{align*}
\Delta C_{1}^{B_{s}} & =-C_{b s}^{b s} \\
\Delta \tilde{C}_{1}^{B_{s}} & =-\tilde{C}_{b s}^{b s} \\
\Delta \tilde{C}_{3}^{B_{s}} & =-\frac{1}{2}\left(D_{b s}^{b s}+\tilde{D}_{b s}^{b s}\right)=-\tilde{D}_{b s}^{b s} . \tag{2.35}
\end{align*}
$$

As in the $b \rightarrow s \bar{q} q$ transitions, the $Z^{\prime}$ effects only correct the Wilson coefficients of the color-allowed operators at a higher loop level, so we will not consider them further here.

To summarize, in table (3) we classify the tree-level $Z^{\prime}$ contributions to the $b \rightarrow s$ transitions according to whether they are relevant or irrelevant to the SM operators.

Before considering the general parameter space, it is worthwhile to consider a few special limits: (1) the LR limit: $\left|B_{b s}^{L}\right|=\left|B_{b s}^{R}\right|, \phi_{b s}^{L}=\phi_{b s}^{R} ;(2)$ the LL limit: $\epsilon^{\psi_{R}} \propto I$; and (3) the RR limit: $\epsilon^{\psi_{L}} \propto I$, where $I$ is the identity. The $Z^{\prime}$ corrections to the Wilson coefficients in these limits are summarized as follows:

1. LR limit: $B_{b s}^{L}=B_{b s}^{R}$.

$$
\Delta C_{1}^{B_{s}}=\Delta \tilde{C}_{1}^{B_{s}}=\Delta \tilde{C}_{3}^{B_{s}}=-\left(B_{b s}^{L}\right)^{2}
$$

|  | SM operators | Beyond SM Operators |
| :---: | :---: | :---: |
| $b \rightarrow s \bar{q} q$ | $\Delta C_{a}, a=3,5,7$ | $\Delta \tilde{C}_{a}, a=3,5,9$ |
| $b \rightarrow s \bar{l} l$ | $\Delta C_{a}, a=9 V, 10 A$ | $\Delta \tilde{C}_{a}, a=9 V, 10 A$ |
| $B_{s}-\bar{B}_{s}$ mixing | $\Delta C_{1}^{B_{s}}$ | $\Delta \tilde{C}_{a}^{B_{s}}, a=1,3$ |

Table 3. Classification of the tree-level $Z^{\prime}$ corrections to the Wilson coefficients in the $b \rightarrow s$ transitions.

$$
\begin{align*}
\Delta C_{3} & =\Delta \tilde{C}_{5}=-\frac{2}{V_{t b} V_{t s}^{*}} B_{b s}^{L} B_{d d}^{L}, \\
\Delta \tilde{C}_{3} & =\Delta C_{5}=-\frac{2}{3 V_{t b} V_{t s}^{*}} B_{b s}^{L}\left(B_{u u}^{R}+2 B_{d d}^{R}\right), \\
\Delta C_{7} & =\Delta \tilde{C}_{9}=-\frac{4}{3 V_{t b} V_{t s}^{*}} B_{b s}^{L}\left(B_{u u}^{R}-B_{d d}^{R}\right), \\
\Delta C_{9 V} & =\Delta \tilde{C}_{9 V}=-\frac{2}{V_{t b} V_{t s}^{*}} B_{b s}^{L}\left(B_{l l}^{L}+B_{l l}^{R}\right), \\
\Delta C_{10 A} & =\Delta \tilde{C}_{10 A}=-\frac{2}{V_{t b} V_{t s}^{*}} B_{b s}^{L}\left(-B_{l l}^{L}+B_{l l}^{R}\right) \tag{2.36}
\end{align*}
$$

2. LL limit: $\epsilon^{\psi_{R}} \propto I$.

$$
\begin{align*}
\Delta C_{1}^{B_{s}} & =-\left(B_{b s}^{L}\right)^{2}, \\
\Delta C_{3} & =-\frac{2}{V_{t b} V_{t s}^{*}} B_{b s}^{L} B_{d d}^{L}, \\
\Delta C_{5} & =-\frac{2}{3 V_{t b} V_{t s}^{*}} B_{b s}^{L}\left(B_{u u}^{R}+2 B_{d d}^{R}\right), \\
\Delta C_{7} & =-\frac{4}{3 V_{t b} V_{t s}^{*}} B_{b s}^{L}\left(B_{u u}^{R}-B_{d d}^{R}\right), \\
\Delta C_{9 V} & =-\frac{2}{V_{t b} V_{t s}^{*}} B_{b s}^{L}\left(B_{l l}^{L}+B_{l l}^{R}\right), \\
\Delta C_{10 A} & =-\frac{2}{V_{t b} V_{t s}^{*}} B_{b s}^{L}\left(-B_{l l}^{L}+B_{l l}^{R}\right) \tag{2.37}
\end{align*}
$$

3. RR limit: $\epsilon^{\psi_{L}} \propto I$.

$$
\begin{align*}
\Delta \tilde{C}_{1}^{B_{s}} & =-\left(B_{b s}^{R}\right)^{2}, \\
\Delta \tilde{C}_{3} & =-\frac{2}{3 V_{t b} V_{t s}^{*}} B_{b s}^{R}\left(B_{u u}^{R}+2 B_{d d}^{R}\right), \\
\Delta \tilde{C}_{5} & =-\frac{2}{V_{t b} V_{t s}^{*}} B_{b s}^{R} B_{d d}^{L}, \\
\Delta \tilde{C}_{9} & =-\frac{4}{3 V_{t b} V_{t s}^{*}} B_{b s}^{R}\left(B_{u u}^{R}-B_{d d}^{R}\right), \\
\Delta \tilde{C}_{9 V} & =-\frac{2}{V_{t b} V_{t s}^{*}} B_{b s}^{R}\left(B_{l l}^{L}+B_{l l}^{R}\right), \\
\Delta \tilde{C}_{10 A} & =-\frac{2}{V_{t b} V_{t s}^{*}} B_{b s}^{R}\left(-B_{l l}^{L}+B_{l l}^{R}\right) . \tag{2.38}
\end{align*}
$$

We will focus on the correlations between $B_{s}-\bar{B}_{s}$ mixing and the hadronic $B_{d}$ meson decays. For the latter, though $Z^{\prime}$-mediated effects can occur in both the QCD and EW penguins, we make a conservative assumption in this paper that they are mainly manifest in the EW penguins, such that $\left|\Delta C_{3,5}\right| \ll\left|\Delta C_{7}\right|$, as suggested in [20, 22]. With this restriction, there are only three relevant parameters for each special limit: the modulus of $B_{b s}^{L}$ (or $B_{b s}^{R}$ ), its phase $\phi_{b s}^{L}$ (or $\phi_{b s}^{R}$ ), and the real $B_{d d}^{R}\left(\simeq-B_{u u}^{R} / 2\right)$. These parameters need to satisfy

$$
\begin{equation*}
\left|B_{b s}^{L}\right|<\left|B_{d d}^{L}\right| \ll\left|B_{d d}^{R}\right| \tag{2.39}
\end{equation*}
$$

in the LR and LL limits, and

$$
\begin{equation*}
\left|B_{b s}^{R}\right|<\left|B_{d d}^{R}\right|, \quad\left|B_{d d}^{L}\right| \ll\left|B_{d d}^{R}\right| \tag{2.40}
\end{equation*}
$$

in the RR limit. Here $\left|B_{b s}^{L, R}\right|<\left|B_{d d}^{L, R}\right|$ is due to the fact that, under the assumption of small fermion mixing angles, the modulous of off-diagonal elements in the coupling matrix in eq. (2.10) should be smaller than that of diagonal ones; $\left|B_{d d}^{L}\right| \ll\left|B_{d d}^{R}\right|$ is due to $\left|\Delta C_{3,5}\right| \ll\left|\Delta C_{7}\right|$. Later in the paper, we will consider the more general parameter space for the $Z^{\prime}$-mediated effects in the EW penguins, which has five free parameters: $\left|B_{b s}^{L, R}\right|$, $\phi_{b s}^{L, R}$ and $B_{d d}^{R}$.

### 2.2 Effective couplings at the $b$ mass scale

To achieve sufficient precision for these observables, it is necessary to have an accurate knowledge of the relevant Wilson coefficients at the $b$ quark mass scale $m_{b}=4.2 \mathrm{GeV}$. The Wilson coefficients at the $b$ mass scale can be obtained as follows:

$$
\begin{equation*}
\vec{C}\left(m_{b}\right)=U\left(m_{b}, M_{W}\right) \vec{C}\left(M_{W}\right), \tag{2.41}
\end{equation*}
$$

where $\vec{C}$ is a vector with entries consisting of the Wilson coefficients and $U$ is the evolution matrix. The observables can then be expressed in terms of the Wilson coefficients at the $m_{b}$ scale (for general discussions, see e.g. [23]). All parameter values used in our calculations are summarized in C.

- $\boldsymbol{B}_{\boldsymbol{s}}$ mixing. Following [23], the NP probes $C_{B_{s}}$ and $\phi_{B_{s}}$, which are defined in eq. (1.1), are calculated to be

$$
\begin{equation*}
C_{B_{s}} e^{2 i \phi_{B_{s}}}=1-3.59 \times 10^{5}\left(\Delta C_{1}^{B_{s}}+\Delta \tilde{C}_{1}^{B_{s}}\right)+2.04 \times 10^{6} \Delta \tilde{C}_{3}^{B_{s}} \tag{2.42}
\end{equation*}
$$

at the $m_{b}$ scale. The large coefficients of the correction terms are due to the fact that the NP is introduced at tree-level while the SM limit is a loop-level effect.

- $\boldsymbol{B}_{\boldsymbol{d}} \rightarrow \boldsymbol{\pi} \boldsymbol{K}_{\boldsymbol{S}}$ decays. The $B_{d} \rightarrow \pi K_{S}$ decays have recently received considerable interest in the literature (see e.g. [22, 24-28]). In [22], it is pointed out that a deviation of $\mathcal{S}_{\pi K_{S}}$ from its SM value can be understood as a modification of the ratio

$$
\begin{equation*}
q e^{i \phi}=\frac{P}{T+C}, \tag{2.43}
\end{equation*}
$$

in which $T, C$ and $P$ denote the color-allowed tree, color-suppressed tree, and EW penguin contributions in the decay amplitude, respectively. In the class of models considered here, the family non-universal $Z^{\prime}$ interactions modify $q e^{i \phi}$ through the relation

$$
\begin{equation*}
q e^{i \phi}=0.76\left(1+158.1 \Delta C_{7}-102.4 \Delta \tilde{C}_{9}\right) \tag{2.44}
\end{equation*}
$$

in which $q$ and $\phi$ are given by 0.76 and zero, respectively, in the SM limit.

- $\boldsymbol{B}_{\boldsymbol{d}} \rightarrow\left(\psi, \phi, \boldsymbol{\eta}^{\prime}, \boldsymbol{\rho}, \boldsymbol{\omega}, \boldsymbol{f}^{0}\right) \boldsymbol{K}_{\boldsymbol{S}}$ decays. The direct and the mixing-induced $C P$ asymmetries in the $B_{d}$ hadronic decays are parametrized as follows:

$$
\begin{equation*}
\mathcal{C}_{f_{C P}}=\frac{1-\left|\lambda_{f_{C P}}\right|^{2}}{1+\left|\lambda_{f_{C P}}\right|^{2}}, \quad \mathcal{S}_{f_{C P}}=\frac{2 \operatorname{Im}\left[\lambda_{f_{C P}}\right]}{1+\left|\lambda_{f_{C P}}\right|^{2}} \tag{2.45}
\end{equation*}
$$

In the above, $\lambda_{f_{C P}}$ is defined by

$$
\begin{equation*}
\lambda_{f_{C P}} \equiv-\eta_{f_{C P}} \frac{q_{B_{d}}}{p_{B_{d}}} \frac{\bar{A}_{f_{C P}}}{A_{f_{C P}}} \tag{2.46}
\end{equation*}
$$

with

$$
\begin{equation*}
\left.\frac{q_{B_{d}}}{p_{B_{d}}}\right|_{\Delta \Gamma_{B_{d}}=0}=-\frac{\left(M_{B_{d}}\right)_{12}^{*}}{\left|\left(M_{B_{d}}\right)_{12}\right|}=-e^{-2 i \phi_{B_{d}}} \tag{2.47}
\end{equation*}
$$

Here $q_{B_{d}}$ and $p_{B_{d}}$ are $B_{d}$ mixing coefficients

$$
\begin{align*}
\left|B_{L}\right\rangle & =p_{B_{d}}\left|B_{d}\right\rangle+q_{B_{d}}\left|\bar{B}_{d}\right\rangle \\
\left|B_{H}\right\rangle & =p_{B_{d}}\left|B_{d}\right\rangle-q_{B_{d}}\left|\bar{B}_{d}\right\rangle \tag{2.48}
\end{align*}
$$

$M_{B_{d}}$ is the $B_{d}-\bar{B}_{d}$ mass matrix, and $A_{f_{C P}}$ is the decay amplitude of $B_{d} \rightarrow f_{C P}$ ( $\bar{A}_{f_{C P}}$ is its $C P$ conjugate.).

The SM predicts that $\phi_{B_{d}}=\beta \equiv \arg \left[-\left(V_{c d} V_{c b}^{*}\right) /\left(V_{t d} V_{t b}^{*}\right)\right]$ and that a non-trivial weak phase enters $A_{f_{C P}}$ only at order $\mathcal{O}\left(\lambda^{2}\right)$. Therefore, for time-dependent decays proceeding via $b \rightarrow s \bar{q} q(q=u, d, c, s)$, including $B_{d} \rightarrow \psi K_{S}$ and penguin-dominated modes such as $B_{d} \rightarrow\left(\phi, \eta^{\prime}, \pi, \rho, \omega, f^{0}\right) K_{S}$, the relations in eq. (1.4) are obtained.

However, these results are greatly changed with the involvement of family non-universal $Z^{\prime}$ bosons, since this allows for a new weak phase to enter $A_{f_{C P}}$ at tree level. Following Ali et. al. [29], the $\lambda_{f_{C P}}$ parameters of $B_{d} \rightarrow\left(\psi, \phi, \eta^{\prime}, \pi, \rho, \omega, f^{0}\right) K_{S}$ are given by

$$
\begin{align*}
\lambda_{\psi K_{S}}= & (-0.63+0.74 i)  \tag{2.49}\\
& \frac{1+(0.18-0.01 i)\left(\Delta C_{7}+\Delta \tilde{C}_{7}\right)^{*}-(0.06-0.04 i)\left(\Delta C_{9}+\Delta \tilde{C}_{9}\right)^{*}}{1+(0.17+0.01 i)\left(\Delta C_{7}+\Delta \tilde{C}_{7}\right)-(0.05+0.05 i)\left(\Delta C_{9}+\Delta \tilde{C}_{9}\right)} \\
\lambda_{\phi K_{S}}= & (-0.70+0.70 i)  \tag{2.50}\\
& \frac{1+(14.57+5.88 i)\left(\Delta C_{7}+\Delta \tilde{C}_{7}\right)^{*}+(15.08+5.92 i)\left(\Delta C_{9}+\Delta \tilde{C}_{9}\right)^{*}}{1+(14.39+5.64 i)\left(\Delta C_{7}+\Delta \tilde{C}_{7}\right)+(14.90+5.66 i)\left(\Delta C_{9}+\Delta \tilde{C}_{9}\right)}
\end{align*}
$$



Figure 1. Correlated constraints on $\left|B_{b s}^{L}\right|$ and $\phi_{b s}^{L}$ are presented. In these two panels, random values for $C_{B_{s}}$ and $\phi_{B_{s}}^{\mathrm{NP}}$ from the experimentally allowed regions (see table 1) are mapped to the $\left|B_{b s}^{L}\right|-\phi_{b s}^{L}$ plane using eq. (3.1), with an assumed $25 \%$ uncertainty (a typical value from nonperturbative effects) assumed for the coefficients. The left (right) panel is the LR (LL) limit.

$$
\begin{align*}
\lambda_{\eta^{\prime} K_{S}}= & (-0.70+0.69 i)  \tag{2.51}\\
& \frac{1+(2.11+0.67 i)\left(\Delta C_{7}+\Delta \tilde{C}_{7}\right)^{*}+(2.10+0.54 i)\left(\Delta C_{9}+\Delta \tilde{C}_{9}\right)^{*}}{1+(2.08+0.65 i)\left(\Delta C_{7}+\Delta \tilde{C}_{7}\right)+(2.07+0.52 i)\left(\Delta C_{9}+\Delta \tilde{C}_{9}\right)} \\
\lambda_{\rho K_{S}}= & (-0.74+0.66 i)  \tag{2.52}\\
& \frac{1-(38.75+3.29 i)\left(\Delta C_{7}+\Delta \tilde{C}_{7}\right)^{*}-(47.95+4.11 i)\left(\Delta C_{9}+\Delta \tilde{C}_{9}\right)^{*}}{1-(38.11+5.23 i)\left(\Delta C_{7}+\Delta \tilde{C}_{7}\right)-(47.15+6.50 i)\left(\Delta C_{9}+\Delta \tilde{C}_{9}\right)} \\
\lambda_{\omega K_{S}}= & (-0.71+0.70 i)  \tag{2.53}\\
& \frac{1+(31.97+4.76 i)\left(\Delta C_{7}+\Delta \tilde{C}_{7}\right)^{*}+(18.84+2.75 i)\left(\Delta C_{9}+\Delta \tilde{C}_{9}\right)^{*}}{1+(31.81+4.67 i)\left(\Delta C_{7}+\Delta \tilde{C}_{7}\right)+(18.74+2.70 i)\left(\Delta C_{9}+\Delta \tilde{C}_{9}\right)} \\
\lambda_{f^{0} K_{S}}= & (-0.70+0.70 i)  \tag{2.54}\\
& \frac{1+(3.19+0.93 i)\left(\Delta C_{7}+\Delta \tilde{C}_{7}\right)^{*}-(0.12+0.15 i)\left(\Delta C_{9}+\Delta \tilde{C}_{9}\right)^{*}}{1+(3.16+0.90 i)\left(\Delta C_{7}+\Delta \tilde{C}_{7}\right)-(0.12+0.15 i)\left(\Delta C_{9}+\Delta \tilde{C}_{9}\right)}
\end{align*}
$$

In contrast to the $B_{d} \rightarrow \psi K_{S}$ decay, in which the NP effects are suppressed by the SM treelevel contribution, family non-universal $\mathrm{U}(1)^{\prime}$ couplings indeed result in sizable corrections to $\lambda_{f_{C P}}$ for the penguin-dominated modes.

## 3 Results and analysis

### 3.1 Correlated analysis (I). Special limits

In this section, we will present a correlated analysis of the $\Delta B=1,2$ processes which occur via $b \rightarrow s$ transitions, focusing first on the special limits of the parameter space as presented in section 2.1. As the physics of the RR limit is very similar to that of the LL limit, we focus in this paper on the LR and LL limits as representative examples.

We first consider the constraints on this class of family non-universal $\mathrm{U}(1)^{\prime}$ scenarios which arise from $B_{s}-\bar{B}_{s}$ mixing. With the renormalization scale chosen as the $b$-quark


Figure 2. The constraints on $B_{d d}^{R}$ from $q e^{i \phi}$ are shown. The points from the $\left|B_{b s}^{L}\right|-\phi_{b s}^{L}$ plane (see figure 1) are randomly combined with scattered points of $B_{d d}^{R}\left(10^{-3} \leq\left|B_{d d}^{R}\right| \leq 10^{-1}\right)$ and then mapped to the $q \cos \phi-q \sin \phi$ plane according to eq. (3.6). The colors of the points in this plane indicate the C.L. that their inverse images represent in figure 1. The two dashed lines specify the experimentally allowed ranges that result from the $\chi^{2}$ fit of the $B \rightarrow \pi K$ (and $B \rightarrow \pi \pi$ ) data at $1 \sigma$ and $90 \%(\simeq 1.7 \sigma)$ C.L., respectively [24]. The left (right) panels are the LR (LL) limits.


Figure 3. The time-dependent $C P$ asymmetries of the charmed $B_{d} \rightarrow \psi K_{S}$ decay are presented (with $\left|V_{u b}\right|=3.51 \times 10^{-3}$, as is used in the SM calculation [33]). The box is at $1 \sigma$ C.L. and the dark point is the SM limit. The left (right) panels are the LR (LL) limits.
mass, $m_{b}=4.2 \mathrm{GeV}$, the NP probes $C_{B_{s}}$ and $\phi_{B_{s}}^{\mathrm{NP}}$ are given by

$$
\begin{align*}
& C_{B_{s}} e^{2 i \phi_{B_{s}}^{\mathrm{NP}}}=1+1.32 \times 10^{6} \Delta C_{1}^{B_{s}} \\
& C_{B_{s}} e^{2 i \phi_{B_{s}}}=1-3.59 \times 10^{5} \Delta \tilde{C}_{1}^{B_{s}} \tag{3.1}
\end{align*}
$$

in the LR and LL limits, respectively. These conditions involve two of the three free parameters of each limit: $\left|B_{b s}^{L}\right|$ and $\phi_{b s}^{L}$. The experimental constraints on these parameters from $B_{s}-\bar{B}_{s}$ mixing are illustrated in figure 1. The left panel corresponds to the LR limit and the right one corresponds to the LL limit; in this section we will present the results for these two limits together, so that it is easy to make comparisons between the two cases. In each case, there are two separate shaded regions, corresponding to the two $\phi_{B_{s}}^{\mathrm{NP}}$ solutions (see table 1). For each region, the various colors of the points specify the different confidence levels (C.L.) of the relevant $C_{B_{s}}$ and $\phi_{B_{s}}^{\mathrm{NP}}$ values. To explain the


Figure 4. With $B_{b s}^{L}$ and $B_{d d}^{R}$ constrained by $B_{s}-\bar{B}_{s}$ mixing and $B_{d} \rightarrow \pi K_{S}$, the NP contributions to $\mathcal{C}_{\left(\phi, \eta^{\prime}, \rho, \omega, f_{0}\right) K_{S}}$ and $\mathcal{S}_{\left(\phi, \eta^{\prime}, \rho, \omega, f_{0}\right) K_{S}}$ are shown. The left (right) panels are the LR (LL) limits. The colors specify the C.L. that their inverse image points represent in figures 1 and 2 (yellow for $1 \sigma$ C.L. and blue for $2 \sigma$ and $1.7 \sigma$ C.L.). The boxes specify the experimentally allowed regions at $1 \sigma$ and $1.7 \sigma$, and the dark point denotes the SM limit.


Figure 5. The allowed $\left|B_{b s}^{L}\right|, \phi_{b s}^{L}\left[^{\circ}\right]$ and $B_{d d}^{R}$ are shown. They are constrained from $B_{s}-\bar{B}_{s}$ mixing (at $2 \sigma$ C.L.) and the $\chi^{2}$ fit of the $B \rightarrow \pi K$ (and $B \rightarrow \pi \pi$ ) data (at $1 \sigma$ C.L.), then selected by $\mathcal{C}_{\left(\phi, \eta^{\prime}, \rho, \omega, f_{0}\right) K_{S}}, \mathcal{S}_{\left(\phi, \eta^{\prime}, \rho, \omega, f_{0}\right) K_{S}}$ (at 1.7 $\sigma$ C.L. for the first four panels and $1.5 \sigma$ C.L. for the others). The left (right) panels are the LR (LL) limits.


Figure 6. The points in figure 5 are inversely mapped to the $q \cos \phi-q \sin \phi$ plane. The panels in the first row correspond to a parameter selection from $\mathcal{C}_{\left(\phi, \eta^{\prime}, \rho, \omega, f_{0}\right) K_{S}}, \mathcal{S}_{\left(\phi, \eta^{\prime}, \rho, \omega, f_{0}\right) K_{S}}$ at $1.7 \sigma$ C.L. in figure 5 , and those in the second row at $1.5 \sigma$ C.L.. The left (right) panels are the LR (LL) limits.
observed discrepancy in $B_{s}-\bar{B}_{s}$ mixing from the SM prediction, $\left|B_{b s}^{L}\right|$ is required to be $\sim 10^{-3}$. This reflects two facts: (1) unlike $\phi_{B_{s}}^{\mathrm{NP}}$, the modulus $C_{B_{s}}$ does not deviate from its SM prediction significantly (the experimental value of $C_{B_{s}}$ has an at most $\mathcal{O}(1)$ shift from its SM prediction at $2 \sigma$ C.L.); (2) the $Z^{\prime}$ corrections are from tree level, and hence can easily explain this small deviation (only a small coupling is necessary, according to eq. (3.1) and eq. (2.36), eq. (2.37)). The smallness of $\left|B_{b s}^{L}\right|$ is generically consistent with our assumption of small fermion mixing angles, since $B_{b s}^{L}$ is proportional to them as well as to $g_{2} M_{Z} /\left(g_{1} M_{Z^{\prime}}\right)$ (see eq. (2.14) and the comments under eq. (2.21)). In addition, due to the smallness of $\left|B_{b s}^{L}\right|$, the experimental constraints from the branching ratio $\operatorname{Br}\left(B_{s} \rightarrow \mu^{+} \mu^{-}\right)$ can be easily satisfied. For details, see B.

Before we move to the other $b \rightarrow s$ processes, we have some comments on the influence of $B_{s}-\bar{B}_{s}$ mixing on another FCNC $Z^{\prime}$ coupling $B_{b d}^{L, R}$. In the SM, the mass differences of $B_{d}$ and $B_{s}$ mesons are predicted to be (e.g., see [30])

$$
\begin{align*}
& \Delta M_{d}^{\mathrm{SM}}=(0.53 \pm 0.02)\left(\frac{\left|V_{t d}\right|}{0.0082}\right)^{2}\left(\frac{f_{B_{d}}}{200 \mathrm{MeV}}\right)^{2} \frac{B}{0.85} \mathrm{ps}^{-1} \\
& \Delta M_{s}^{\mathrm{SM}}=(19.3 \pm 0.6)\left(\frac{\left|V_{t s}\right|}{0.00405}\right)^{2}\left(\frac{f_{B_{s}}}{240 \mathrm{MeV}}\right)^{2} \frac{B}{0.85} \mathrm{ps}^{-1} \tag{3.2}
\end{align*}
$$

with $f_{B_{d}, B_{s}}$ being decay constant of $B_{d}\left(B_{s}\right)$ and $B$ being bag factor. Comparing with the
experimental data [31]

$$
\begin{align*}
& \Delta M_{d}=\Delta M_{d}^{\mathrm{SM}}\left(1+\frac{\Delta M_{d}^{\mathrm{NP}}}{\Delta M_{d}^{\mathrm{SM}}}\right)=0.507 \pm 0.005 \mathrm{ps}^{-1} \\
& \Delta M_{s}=\Delta M_{s}^{\mathrm{SM}}\left(1+\frac{\Delta M_{s}^{\mathrm{NP}}}{\Delta M_{s}^{\mathrm{SM}}}\right)=17.77 \pm 0.12 \mathrm{ps}^{-1} \tag{3.3}
\end{align*}
$$

we have the relation

$$
\begin{equation*}
\frac{\Delta M_{d}^{\mathrm{NP}}}{\Delta M_{d}^{\mathrm{SM}}} \sim \frac{\Delta M_{s}^{\mathrm{NP}}}{\Delta M_{s}^{\mathrm{SM}}} \tag{3.4}
\end{equation*}
$$

with

$$
\begin{equation*}
\frac{\Delta M_{d}^{\mathrm{NP}}}{\Delta M_{s}^{\mathrm{NP}}} \sim \frac{\left|B_{b d}^{L, R}\right|^{2}}{\left|B_{b s}^{L, R}\right|^{2}}, \quad \frac{\Delta M_{d}^{\mathrm{SM}}}{\Delta M_{s}^{\mathrm{SM}}} \sim \lambda^{2} \approx 0.04 \tag{3.5}
\end{equation*}
$$

But, according to eq. (2.14), the modulous of $B_{b d}^{L, R}$ usually is comparable with that of $B_{b s}^{L, R}$, a fine-tuning of $\mathcal{O}(10 \%)$ level therefore is needed in $\left|B_{b d}^{L, R}\right|$ to satisfy the experimental constraints from $\Delta M_{d}$. In this paper, we will work under the assumption of negligible $\left|B_{b d}^{L, R}\right|$, and therefore will neglect its possible effect in the NP observables.

The second process of interest is $B_{d} \rightarrow \pi K_{S}$. The time-dependent $C P$ asymmetries of these decays can be sizably affected by NP, as has been pointed out in [22]. The experimental constraints on $q e^{i \phi}$ (defined in eq. (2.43)) for different C.L.'s from the $B \rightarrow$ $\pi K$ (and the $B \rightarrow \pi \pi$ ) data have previously been obtained in [24]. In figure 2, we illustrate how $B_{d d}^{R}$ is constrained through $q e^{i \phi}$, using the parameter values of $\left|B_{b s}^{L}\right|$ and $\phi_{b s}^{L}$ obtained in figure 1, along with the following relations:

$$
\begin{align*}
q e^{i \phi} & =0.76\left(1+55.7 \Delta C_{7}\right) \\
q e^{i \phi} & =0.76\left(1+158.1 \Delta C_{7}\right) \tag{3.6}
\end{align*}
$$

which are valid in the LR and LL limits, respectively. There are two distribution regions which are specified by different colors in each panel, again due to the two $\phi_{B_{s}}^{\mathrm{NP}}$ solutions. Note that in the LL limit, the shaded region passes through both of the minimal points that were found in the $\chi^{2}$ fit of the $B \rightarrow \pi K$ and $B \rightarrow \pi \pi$ data in [24].

In the scenarios under consideration, the constraints from $B_{s}-\bar{B}_{s}$ mixing and $B_{d} \rightarrow$ $\pi K_{S}$ decays place bounds on each of the three free parameters $\left|B_{b s}^{L}\right|, \phi_{b s}^{L}$ and $B_{d d}^{R}$. The natural question is then whether the experimentally allowed values for these parameters also satisfy the constraints resulting from the possibly anomalous values of $\Delta \mathcal{C}_{f_{C P}}$ and $\Delta \mathcal{S}_{f_{C P}}$ in the remaining penguin-dominated $B_{d} \rightarrow\left(\phi, \eta^{\prime}, \rho, \omega, f_{0}\right) K_{S}$ decays. To address this issue, we assume a $15 \%$ uncertainty in the SM calculations for each of these modes (as well as for the $B_{d} \rightarrow \pi K_{S}$ mode) and a $25 \%$ uncertainty for the NP contributions. Here $15 \%$ is a typical uncertainty level for the hadronic matrix elements of the SM FC operators (e.g., see [32]) and is also the least necessary one to explain the experimental data of $\mathcal{C}_{\psi K_{S}}$ and $\mathcal{S}_{\psi K_{S}}$ in the SM (see figure 3 where the NP effects are negligible). As for the difference of the uncertainty levels between the SM and NP calculations, it is caused by the fact that
the hadronic matrix elements of the FC operators in the SM are better understood than they are for the NP operators. In figure 4, we systematically illustrate the time-dependent $C P$ asymmetries in the penguin-dominated modes, using the parameter values obtained above and eqs. (2.49)-(2.54). For these modes with the exception of $B_{d} \rightarrow \rho K_{S}$, there are $0.5 \sim 2 \sigma$ deviations for $\mathcal{C}_{f_{C P}}, \mathcal{S}_{f_{C P}}$, or both. Though our model only induces negligible effects on $B_{d}-\bar{B}_{d}$ mixing under the assumption of small $\left|B_{b d}\right|$, due to the interference effects between the $B_{d}-\bar{B}_{d}$ mixing phase and $\phi_{b s}^{L}$ which affects the decay asymmetries $\frac{\bar{A}_{C P}}{A_{f_{C P}}}$ in eq. (2.46), the points in figure 4 are scattered away from the SM limits. This results in a dispersion such that there are always some points lying in the $1 \sigma$ region for each of these modes.

To show that all of the constraints can be satisfied simultaneously, we have carried out a correlated analysis among the $B_{s}-\bar{B}_{s}$ mixing and the $B_{d} \rightarrow\left(\pi, \phi, \eta^{\prime}, \rho, \omega, f_{0}\right) K_{S}$ $C P$ asymmetries. The allowed values for $\left|B_{b s}^{L}\right|, \phi_{b s}^{L}$ and $B_{d d}^{R}$ in the LR and LL limits are illustrated in figure 5 . We see that in figure 5 there indeed exist parameter regions where the anomalies in $B_{s}-\bar{B}_{s}$ mixing and the time-dependent $C P$ asymmetries of $B_{d} \rightarrow$ $\left(\pi, \phi, \eta^{\prime}, \rho, \omega, f_{0}\right) K_{S}$, can be explained by NP at reasonable C.L.. The allowed $\left|B_{b s}^{R}\right|$ and $\phi_{b s}^{L}$ values can explain both solutions of $B_{s}-\bar{B}_{s}$ mixing phase; and the allowed $\left|B_{d d}^{R}\right|$ values vary from 0.08 to smaller values. If we want to get a better fit for $\mathcal{C}_{\left(\pi, \phi, \eta^{\prime}, \rho, \omega, f_{0}\right) K_{S}}$ and $\mathcal{S}_{\left(\pi, \phi, \eta^{\prime}, \rho, \omega, f_{0}\right) K_{S}},\left|B_{d d}^{R}\right| \gtrsim 10^{-2}$ is typically required. To see this point, we take for example $\mathcal{C}_{\pi K_{S}}$ and $\mathcal{S}_{\pi K_{S}}$, and map the points in figure 5 back to the $q \cos \phi-q \sin \phi$ plane, as illustrated in Fig 6. In this figure we see that the points with $B_{d d}^{R}<-0.01$ are closer to the minima of the $\chi^{2}$ fit of the $B \rightarrow \pi K$ and $B \rightarrow \pi \pi$ data, leading to a better fit compared to the one obtained in the SM limit. Therefore, $\left|B_{d d}^{R}\right| \gtrsim 10^{-2}$ is important in improving the agreement with experimental data in the penguin-dominated $B_{d}$ decays. ${ }^{5}$

The favored parameter values for $\left|B_{d d}^{R}\right|$ are interesting for collider detection. For $\left(V_{d_{R}} \tilde{\epsilon}^{d_{R}} V_{d_{R}}\right)_{11} \sim \mathcal{O}(1)$, this implies that $g_{1} M_{Z^{\prime}} /\left(g_{2} M_{Z}\right) \sim 10-100$ or a TeV scale $Z^{\prime}$ boson for $g_{2} \lesssim g_{1}$, a range approachable at the LHC (e.g., see [17, 34]). This fact is also important for the effective Lagrangian in eq. (2.16) which is obtained by integrating out the $Z^{\prime}$ boson. While applying it to our analysis, we neglected the effects of the renormalization group running between $Z^{\prime}$ mass scale and EW scale, which is justified only for a small gap between these two scale or for a low-scale $Z^{\prime}$ boson. In addition, we emphasize that the favored parameter regions are consistent with our assumption that the non-universal $Z^{\prime}$ effects in QCD penguins are negligible. This assumption requires $\left|\Delta C_{3,5}\right| \ll\left|\Delta C_{7}\right|$ or $\left|B_{b s}^{L}\right|<\left|B_{d d}^{L}\right| \ll\left|B_{d d}^{R}\right|$. Since $\left|B_{b s}^{L}\right|$ and $\left|B_{d d}^{R}\right|$ are favored to be $\sim 10^{-3}$ and $\gtrsim 10^{-2}$ respectively, this relation can be easily accommodated. At last, to implement our discussions, we take a $\chi^{2}$ fit in the SM and in the non-universal $\mathrm{U}(1)^{\prime}$ models for all relevant observables except $q e^{i \phi}$. We find that the reduced $\chi^{2}$ value (i.e., $\chi^{2} /$ D.O.F.) in the SM is larger than 2 , and that of the best fit in both LL and LR limits in the $\mathrm{U}(1)^{\prime}$ models is smaller than 1 . Therefore, a better fit is obtained in the latter.

[^4]

Figure 7. The distributions of $\left|B_{b s}^{L, R}\right|$ and $\phi_{b s}^{L, R}$ resulting from $B_{s}-\bar{B}_{s}$ mixing constraints are shown. The blue and purple points can be mapped to the experimentally allowed $\left\{C_{B_{s}}, \phi_{B_{s}}^{\mathrm{NP}}\right\}$ regions with $1 \sigma$ and $2 \sigma$ C.L., respectively. Here we did not distinct S 1 and S 2 solutions any more.


Figure 8. The constraints on $B_{d d}^{R}$ from $q e^{i \phi}$ are illustrated. The points of $\left|B_{b s}^{L, R}\right|$ and $\phi_{b s}^{L, R}$ from figure 7 are randomly combined with the scattered points of $B_{d d}^{R}\left(10^{-3}<\left|B_{d d}^{R}\right|<10^{-1}\right)$ and then mapped to the $q \cos \phi-q \sin \phi$ plane according to eq. (2.44). The colors of the points in this plane indicate the C.L. that their inverse images represent in figure 7 .


Figure 9. With the values of $\left|B_{b s}^{L, R}\right|, \phi_{b s}^{L, R}$ and $B_{d d}^{R}$ fixed by $B_{s}-\bar{B}_{s}$ mixing and $B_{d} \rightarrow \pi K_{S}$ decay, the NP contributions to $\mathcal{C}_{\left(\phi, \eta^{\prime}, \rho, \omega, f_{0}\right) K_{S}}$ and $\mathcal{S}_{\left(\phi, \eta^{\prime}, \rho, \omega, f_{0}\right) K_{S}}$ are illustrated in the first five panels. The colors of the points specify the C.L. that their inverse image points represent in figure 7 and figure 8 (yellow denotes $1 \sigma$ C.L. in both and blue denotes $2 \sigma$ and 1.7 $\sigma$ C.L., separately). In the last panel, the $C P$ asymmetries of the charmed $B_{d} \rightarrow \psi K_{S}$ decay are presented $\left(\left|V_{u b}\right|=3.51 \times 10^{-3}\right.$ [33]). For each, the two boxes specify the $1 \sigma$ and $1.7 \sigma$ allowed regions (except for the last panel, where only the $1 \sigma$ box is given), and the dark point denotes the SM limit.

### 3.2 Correlated analysis (II). General case

As discussed in section 2.1, there are five free parameters in the general case: $\left|B_{b s}^{L, R}\right|, \phi_{b s}^{L, R}$, and $B_{d d}^{R}$. Let us focus first on $B_{s}-\bar{B}_{s}$ mixing again. The general relation in eq. (2.42) involves four of the five free parameters: $\left|B_{b s}^{L, R}\right|$ and $\phi_{b s}^{L, R}$. In figure 7, we show how the experimentally allowed parameter values are distributed. For each of the two top panels in figure 7, there are two peaks and two valleys toward the right. The two peaks in the left panel correspond to the LL limit, and the two in the right panel correspond to the


Figure 10. The distributions of $\left|B_{b s}^{L, R}\right|, \phi_{b s}^{L, R}$ and $B_{d d}^{R}$ are shown. The values are constrained by $B_{s}-\bar{B}_{s}$ mixing ( $2 \sigma$ C.L.) and the $\chi^{2}$ fit of the $B \rightarrow \pi K$ (and $B \rightarrow \pi \pi$ ) data ( $1 \sigma$ C.L.), then selected by $\mathcal{C}_{\left(\phi, \eta^{\prime}, \rho, \omega, f_{0}\right) K_{S}}, \mathcal{S}_{\left(\phi, \eta^{\prime}, \rho, \omega, f_{0}\right) K_{S}}(1.7 \sigma$ C.L. for the first four panels and 1.5 $\sigma$ C.L. for the rest).


Figure 11．The points in figure 11 are inversely mapped to the $q \cos \phi-q \sin \phi$ plane．The parameter selection from $\mathcal{C}_{\left(\phi, \eta^{\prime}, \rho, \omega, f_{0}\right) K_{S}}, \mathcal{S}_{\left(\phi, \eta^{\prime}, \rho, \omega, f_{0}\right) K_{S}}$ is shown at 1．7 $\sigma$ C．L．in figure 11 the left panel，and at $1.5 \sigma$ C．L．in the right panel．

RR limit．Meanwhile，the points in the right panel which are associated with the LL limit and the points in the left panel which are associated with the RR limit are localized in the regions $\left|B_{b s}^{L}\right| \approx 0$ and $\left|B_{b s}^{R}\right| \approx 0$ ，respectively．As for the valleys in both panels，they correspond to the LR limit．Observe that there are two solutions for each of these three limits which are specified by a difference of $180^{\circ}$ either in $\phi_{b s}^{L}$ or $\phi_{b s}^{R}$（or both）．We only showed one of the two solutions in figure 1，since the difference between these two solutions can be resolved into $B_{d d}^{R}$ as a minus sign．For the two bottom panels，the three special limits LL，LR and RR correspond to the bottom boundary，the diagonal line（the one from left－bottom to right－up）and the left boundary in the left one，respectively．Clearly，for the LL（RR）limit，$\left|B_{b s}^{L}\right|\left(\left|B_{b s}^{R}\right|\right)$ has a relatively large value compared to the one in the LR limit，as seen in figure 1．In the right panel，these three limits correspond to the two parallel bands $\phi_{b s}^{L} \sim-50^{\circ}, 130^{\circ}$ ，the diagonal line（the one from left－bottom to right－up） and the two parallel bands $\phi_{b s}^{R} \sim-50^{\circ}, 130^{\circ}$ ，respectively．

In figure 8，we illustrate how the yet free parameter $B_{d d}^{R}$ is constrained through $q e^{i \phi}$ ， using the parameter values of $\left|B_{b s}^{L, R}\right|$ and $\phi_{b s}^{L, R}$ obtained in figure 7，the relation eq．（2．44）， and the allowed range for $q e^{i \phi}$ as determined in［24］．Compare this figure with figure 2 we see that more points are associated with the solution＂S1＂of $B_{s}-\bar{B}_{s}$ mixing．To see the reason，let us rewrite eq．（2．42）as

$$
\begin{align*}
\frac{A_{s}^{\mathrm{NP}}}{A_{s}^{S M}} e^{2 i \phi_{s}^{\mathrm{NP}}}= & 3.59 \times 10^{5}\left(\left|B_{b s}^{L}\right|^{2} e^{2 i \phi_{b s}^{L}}+\left|B_{b s}^{R}\right|^{2} e^{2 i \phi_{b s}^{R}}\right) \\
& -2.04 \times 10^{6}\left|B_{b s}^{L} B_{b s}^{R}\right| e^{i\left(\phi_{b s}^{L}+\phi_{b s}^{R}\right)} \tag{3.7}
\end{align*}
$$

by using the relation eq．（1．2）．Since the range of the solution＂S1＂is much larger than that of the solution＂S2＂at the same C．L．（see table 1），it can be understood that there are more points in the parameter space corresponding to the＂S1＂solution under the assumption of flat distribution for the random values of the relevant parameters．

We illustrate the time－dependent $C P$ asymmetries of the penguin－dominated modes in figure 9，taking values of $\left|B_{b s}^{L, R}\right|, \phi_{b s}^{L, R}$ and $B_{d d}^{R}$ that are consistent with the constraints from $B_{s}-\bar{B}_{s}$ mixing and $B_{d} \rightarrow \pi K_{S}$ decays．As before，because of interference effects
between the $B_{d}-\bar{B}_{d}$ mixing phase and $\phi_{b s}^{L}$, the points in figure 9 are scattered away from the SM limits. Hence, for each decay mode there are once again always some points lying in the $1 \sigma$ region. The allowed values for $\left|B_{b s}^{L, R}\right|, \phi_{b s}^{L, R}$ and $B_{d d}^{R}$ due to the correlated analysis of the $B_{s}-\bar{B}_{s}$ mixing and the $B_{d} \rightarrow\left(\pi, \phi, \eta^{\prime}, \rho, \omega, f_{0}\right) K_{S} C P$ asymmetries are illustrated in figure 10. In figure 11, as done before, we map the points in figure 10 back to the $q \cos \phi-q \sin \phi$ plane. It is straightforward to see that the points with $B_{d d}^{R}<-0.01$ are closer to the minima of the $\chi^{2}$ fit of the $B \rightarrow \pi K$ and $B \rightarrow \pi \pi$ data, resulting in a better fit than that obtained in the SM limit.

## 4 Conclusions

In this paper, we have studied the constraints on extensions of the SM with family nonuniversal $\mathrm{U}(1)^{\prime}$ gauge symmetries which result from FCNC effects in the $b \rightarrow s$ transitions. Using a model-independent approach in which the main requirements are family universal charges for the first and second generations and small fermion mixing angles, we have performed a correlated analysis of this set of $\Delta B=1,2$ processes. Our results show that within this class of models, the possible anomalies in $B_{s}-\bar{B}_{s}$ mixing and the timedependent $C P$ asymmetries of the penguin-dominated $B_{d} \rightarrow\left(\pi, \phi, \eta^{\prime}, \rho, \omega, f_{0}\right) K_{S}$ decays can be accommodated in a consistent way.

Furthermore, the constraints from $B_{s}-\bar{B}_{s}$ mixing may have nontrivial implications not only for the hadronic decays, but also for the leptonic or semi-leptonic decays of the $B_{d}$ mesons, as discussed in section 2.1. As an example, recent results from BaBar [35] indicate an unexpectedly large isospin asymmetry in the low dilepton mass squared region for combined $B_{d} \rightarrow K l^{+} l^{-}$and $B_{d} \rightarrow K^{*} l^{+} l^{-}$. In addition, $K^{*}$ longitudinal polarization and lepton forward-backward asymmetry are consistent with SM but seem to prefer a wrong-sign $C_{7 \gamma}$ operator. We leave these interesting issues for future exploration.

## Acknowledgments

We thank Cheng-Wei Chiang and Jonathan L. Rosner for useful discussions. Work at ANL is supported in part by the U.S. Department of Energy (DOE), Div. of HEP, Contract DE-AC02-06CH11357. Work at EFI is supported in part by the DOE through Grant No. DE-FG02- 90ER40560. Work at the U. Wisconsin, Madison is supported by the DOE through Grant No. DE-FG02-95ER40896 and the Wisconsin Alumni Research Foundation. T.L. is also supported by the Fermi-McCormick Fellowship. The work of P.L. is supported by the IBM Einstein Fellowship and by NSF grant PHY-0503584.

## A Operators of effective hamiltonians

A complete compilation of the relevant operators for the $b \rightarrow s$ transitions is given in the following:

Current-current operators:

$$
\begin{equation*}
Q_{1}=(\bar{s} u)_{\mathrm{V}-\mathrm{A}}(\bar{u} b)_{\mathrm{V}-\mathrm{A}} \quad Q_{2}=\left(\bar{s}_{\alpha} u_{\beta}\right)_{\mathrm{V}-\mathrm{A}}\left(\bar{u}_{\beta} b_{\alpha}\right)_{\mathrm{V}-\mathrm{A}} \tag{A.1}
\end{equation*}
$$

## QCD-penguin operators:

$$
\begin{array}{rlrl}
Q_{3} & =(\bar{s} b)_{\mathrm{V}-\mathrm{A}} \sum_{q}(\bar{q} q)_{\mathrm{V}-\mathrm{A}} & Q_{4} & =\left(\bar{s}_{\alpha} b_{\beta}\right)_{\mathrm{V}-\mathrm{A}} \sum_{q}\left(\bar{q}_{\beta} q_{\alpha}\right)_{\mathrm{V}-\mathrm{A}} \\
Q_{5} & =(\bar{s} b)_{\mathrm{V}-\mathrm{A}} \sum_{q}(\bar{q} q)_{\mathrm{V}+\mathrm{A}} & Q_{6}=\left(\bar{s}_{\alpha} b_{\beta}\right)_{\mathrm{V}-\mathrm{A}} \sum_{q}\left(\bar{q}_{\beta} q_{\alpha}\right)_{\mathrm{V}+\mathrm{A}} \\
\tilde{Q}_{3} & =(\bar{s} b)_{\mathrm{V}+\mathrm{A}} \sum_{q}(\bar{q} q)_{\mathrm{V}+\mathrm{A}} & \tilde{Q}_{4}=\left(\bar{s}_{\alpha} b_{\beta}\right)_{\mathrm{V}+\mathrm{A}} \sum_{q}\left(\bar{q}_{\beta} q_{\alpha}\right)_{\mathrm{V}+\mathrm{A}} \\
\tilde{Q}_{5} & =(\bar{s} b)_{\mathrm{V}+\mathrm{A}} \sum_{q}(\bar{q} q)_{\mathrm{V}-\mathrm{A}} & \tilde{Q}_{6}=\left(\bar{s}_{\alpha} b_{\beta}\right)_{\mathrm{V}+\mathrm{A}} \sum_{q}\left(\bar{q}_{\beta} q_{\alpha}\right)_{\mathrm{V}-\mathrm{A}} \tag{A.5}
\end{array}
$$

Electroweak penguin operators:

$$
\begin{align*}
Q_{7} & =\frac{3}{2}(\bar{s} b)_{\mathrm{V}-\mathrm{A}} \sum_{q} e_{q}(\bar{q} q)_{\mathrm{V}+\mathrm{A}} & Q_{8} & =\frac{3}{2}\left(\bar{s}_{\alpha} b_{\beta}\right)_{\mathrm{V}-\mathrm{A}} \sum_{q} e_{q}\left(\bar{q}_{\beta} q_{\alpha}\right)_{\mathrm{V}+\mathrm{A}}  \tag{A.6}\\
Q_{9} & =\frac{3}{2}(\bar{s} b)_{\mathrm{V}-\mathrm{A}} \sum_{q} e_{q}(\bar{q} q)_{\mathrm{V}-\mathrm{A}} & Q_{10} & =\frac{3}{2}\left(\bar{s}_{\alpha} b_{\beta}\right)_{\mathrm{V}-\mathrm{A}} \sum_{q} e_{q}\left(\bar{q}_{\beta} q_{\alpha}\right)_{\mathrm{V}-\mathrm{A}}  \tag{A.7}\\
\tilde{Q}_{7} & =\frac{3}{2}(\bar{s} b)_{\mathrm{V}+\mathrm{A}} \sum_{q} e_{q}(\bar{q} q)_{\mathrm{V}-\mathrm{A}} & \tilde{Q}_{8} & =\frac{3}{2}\left(\bar{s}_{\alpha} b_{\beta}\right)_{\mathrm{V}+\mathrm{A}} \sum_{q} e_{q}\left(\bar{q}_{\beta} q_{\alpha}\right)_{\mathrm{V}-\mathrm{A}}  \tag{A.8}\\
\tilde{Q}_{9} & =\frac{3}{2}(\bar{s} b)_{\mathrm{V}+\mathrm{A}} \sum_{q} e_{q}(\bar{q} q)_{\mathrm{V}+\mathrm{A}} & \tilde{Q}_{10} & =\frac{3}{2}\left(\bar{s}_{\alpha} b_{\beta}\right)_{\mathrm{V}+\mathrm{A}} \sum_{q} e_{q}\left(\bar{q}_{\beta} q_{\alpha}\right)_{\mathrm{V}+\mathrm{A}} \tag{A.9}
\end{align*}
$$

## Magnetic penguin operators:

$$
\begin{equation*}
Q_{7 \gamma}=\frac{e}{8 \pi^{2}} m_{b} \bar{s}_{\alpha} \sigma^{\mu \nu}\left(1+\gamma_{5}\right) b_{\alpha} F_{\mu \nu} \quad Q_{8 G}=\frac{g}{8 \pi^{2}} m_{b} \bar{s}_{\alpha} \sigma^{\mu \nu}\left(1+\gamma_{5}\right) T_{\alpha \beta}^{a} b_{\beta} G_{\mu \nu}^{a} \tag{A.10}
\end{equation*}
$$

## Semi-leptonic operators:

$$
\begin{array}{ll}
Q_{9 V}=(\bar{b} s)_{V-A}(\bar{l} l)_{V} & Q_{10 A}=(\bar{b} s)_{V-A}(\bar{l} l)_{A} \\
\tilde{Q}_{9 V}=(\bar{b} s)_{V+A}(\bar{l} l)_{V} & \tilde{Q}_{10 A}=(\bar{b} s)_{V+A}(\overline{l l})_{A} \tag{A.11}
\end{array}
$$

## $B_{s}-\bar{B}_{s}$ mixing operators:

$$
\begin{align*}
Q_{1}^{B_{s}} & =(\bar{s} b)_{V-A}(\bar{s} b)_{V-A} & & Q_{2}^{B_{s}}=\left(\bar{s}_{\alpha} b_{\beta}\right)_{V-A}\left(\bar{s}_{\beta} b_{\alpha}\right)_{V-A} \\
\tilde{Q}_{1}^{B_{s}} & =(\bar{s} b)_{V+A}(\bar{s} b)_{V+A} & & \tilde{Q}_{2}^{B_{s}}=\left(\bar{s}_{\alpha} b_{\beta}\right)_{V+A}\left(\bar{s}_{\beta} b_{\alpha}\right)_{V+A} \\
Q_{3}^{B_{s}}=\tilde{Q}_{3}^{B_{s}} & =(\bar{s} b)_{V+A}(\bar{s} b)_{V-A} & & Q_{4}^{B_{s}}=\tilde{Q}_{4}^{B_{s}}=\left(\bar{s}_{\alpha} b_{\beta}\right)_{V+A}\left(\bar{s}_{\beta} b_{\alpha}\right)_{V-A} \tag{A.12}
\end{align*}
$$

where indices in color singlet currents have been suppressd for simplicity, and $V$ and $A$ refer to $\gamma_{\mu}$ and $\gamma_{\mu} \gamma_{5}$, respectively.

## B Constraints of $\operatorname{Br}\left(B_{s} \rightarrow \mu^{+} \mu^{-}\right)$

For an order of magnitude estimate, one can temporarily ignore the effect of renormalization group running. Then the branching ratio of $B_{s} \rightarrow \mu^{+} \mu^{-}$is given by (also see [36])

$$
\begin{align*}
\operatorname{Br}\left(B_{s} \rightarrow \mu^{+} \mu^{-}\right)= & \tau_{B_{s}} \frac{G_{F}^{2}}{4 \pi} f_{B_{s}}^{2} m_{\mu}^{2} m_{B_{s}} \sqrt{1-\frac{4 m_{\mu}^{2}}{m_{B_{s}}^{2}}}\left|V_{t b}^{*} V_{t s}\right|^{2} \\
& \times\left|\frac{\alpha}{2 \pi \sin ^{2} \theta_{W}} Y\left(\frac{m_{t}^{2}}{M_{W}^{2}}\right)-2 \frac{\left(B_{b s}^{L}+B_{b s}^{R}\right)\left(B_{\mu \mu}^{L}-B_{\mu \mu}^{R}\right)}{V_{t b}^{*} V_{t s}}\right|^{2} \tag{B.1}
\end{align*}
$$

Here $\tau_{B_{s}}$ is the lifetime of $B_{s}$ meson, $f_{B_{s}}$ is the corresponding decay constant, and $Y\left(m_{t}^{2} / M_{W}^{2}\right)$ in the SM part is defined in [37], with

$$
\begin{equation*}
Y(x)=\frac{x}{8}\left(\frac{4-x}{1-x}+\frac{3 x}{(1-x)^{2}} \ln x\right) . \tag{B.2}
\end{equation*}
$$

For $x=m_{t}^{2} / M_{W}^{2}$, we have $Y(x) \sim 1$.
The present experimental exclusion limit at $2 \sigma$ C.L. from a combination of CDF and D0 results is [38]

$$
\begin{equation*}
\operatorname{Br}\left(B_{s} \rightarrow \mu^{+} \mu^{-}\right) \leq 1.5 \times 10^{-7}, \tag{B.3}
\end{equation*}
$$

which thus give a constraint that

$$
\begin{equation*}
\left|3 \times 10^{-3}-\frac{\left(B_{b s}^{L}+B_{b s}^{R}\right)\left(B_{\mu \mu}^{L}-B_{\mu \mu}^{R}\right)}{V_{t b}^{*} V_{t s}}\right|^{2} \lesssim 10^{-4} . \tag{B.4}
\end{equation*}
$$

In the LR and LL limits, we have $B_{b s}^{L}=B_{b s}^{R} \sim 10^{-3}$ and $B_{b s}^{L} \sim 10^{-3}, B_{b s}^{R}=0$, respectively. For $\left.\left(V_{l_{L, R}} \tilde{\epsilon}^{l^{L}, R} V_{l_{L, R}}\right)\right)_{22} \sim \mathcal{O}(1)$ and $g_{1} M_{Z^{\prime}} /\left(g_{2} M_{Z}\right) \sim 10-100$ (a parameter region favored by the correlated analysis of the anomalies in $B_{s}-\bar{B}_{s}$ mixing and time-dependent $C P$ asymmetries of penguin-dominated $B_{d}$ meson decays; for details, see the last paragraph of subsection 3.1), this means that the NP contribution is comparable with the SM one. The experimental bound therefore can be easily satisfied. This conclusion also applies to the general case. This discussion can be easily extended to the $B_{d} \rightarrow K^{(*)} \mu^{+} \mu^{-}$channels (for a model independent discussion, see [39]). Since the amplitudes of the $B_{d} \rightarrow K^{(*)} \mu^{+} \mu^{-}$ decays at the perturbative level are related to the one of $B_{s} \rightarrow \mu^{+} \mu^{-}$by a crossing symmetry, the NP contributions to their branching ratios should be comparable to the SM ones as well. The current experimental bounds on $\operatorname{Br}\left(B_{d} \rightarrow K^{(*)} \mu^{+} \mu^{-}\right)$are still order away from the SM predictions [4], we believe therefore that there is no difficulty to satisfy these constraints in our model.

## C Parameters

The parameters used in our numerical analysis are summarized below:

1. QCD and EW parameters

$$
\begin{align*}
G_{F} & =1.16639 \times 10^{-5} \mathrm{GeV}^{-2}, & \Lambda_{\overline{M S}}^{(5)} & =225 \mathrm{MeV},  \tag{C.1}\\
M_{W} & =80.42 \mathrm{GeV}, & \sin ^{2} \theta_{W} & =0.23,  \tag{C.2}\\
\eta_{2 B} & =0.55, & J_{5} & =1.627,  \tag{C.3}\\
\alpha_{s}\left(M_{Z}\right) & =0.118, & \alpha_{e m} & =1 / 128,  \tag{C.4}\\
\lambda & =0.2252, & A & =0.8117,  \tag{C.5}\\
\bar{\rho} & =0.145, & \bar{\eta} & =0.339,  \tag{C.6}\\
R_{b} & =\sqrt{\rho^{2}+\eta^{2}}=0.378 . & & \tag{C.7}
\end{align*}
$$

2. Masses, Decay Constants, Hadronic Form Factors and Lifetimes

$$
\begin{array}{rlrl}
M_{\pi^{ \pm}} & =0.139 \mathrm{GeV}, & M_{\pi^{0}} & =0.135 \mathrm{GeV}, \\
M_{K} & =0.498 \mathrm{GeV}, & M_{B} & =5.279 \mathrm{GeV}, \\
M_{\phi} & =1.02 \mathrm{GeV}, & M_{\psi} & =2.097 \mathrm{GeV}, \\
M_{\eta^{\prime}} & =0.958 \mathrm{GeV}, & M_{\omega} & =0.783 \mathrm{GeV}, \\
M_{\rho} & =0.776 \mathrm{GeV}, & M_{\eta} & =0.548 \mathrm{GeV} \\
M_{f^{0}} & =0.980 \mathrm{GeV}, & \\
X_{\eta} & =0.57, & Y_{\eta} & =0.82, \\
m_{u}(\mu=4.2 \mathrm{GeV}) & =1.86 \mathrm{MeV}, & m_{d}(\mu=4.2 \mathrm{GeV})=4.22 \mathrm{MeV}, \\
m_{s}(\mu=4.2 \mathrm{GeV}) & =80 \mathrm{MeV}, & m_{c}(\mu=4.2 \mathrm{GeV}) & =0.901 \mathrm{GeV}, \\
m_{b}(\mu=4.2 \mathrm{GeV}) & =4.2 \mathrm{GeV}, & m_{t}^{\text {pole }} & =174 \mathrm{GeV}, \\
f_{\phi} & =237 \mathrm{MeV}, & f_{B} & =190 \mathrm{MeV}, \\
f_{\pi} & =130 \mathrm{MeV}, & f_{K} & =160 \mathrm{MeV}, \\
f_{\psi} & =410 \mathrm{MeV}, & f_{\omega} & =200 \mathrm{MeV}, \\
f_{\rho} & =209 \mathrm{MeV}, & f_{f^{0}} & =180 \mathrm{MeV}, \\
F_{0}^{B \pi}(0) & =0.330, & F_{0}^{B K}(0) & =0.379, \\
F_{1}^{B K}(0) & =0.379, & A_{0}^{B \omega}(0) & =0.280, \\
F_{0}^{B f}(0) & =0.250, & F_{0}^{f K}(0) & =0.030, \\
A_{0}^{B \rho} & =0.280, & f_{B_{s}} \sqrt{\hat{B}_{B_{s}}} & =0.262 \\
\tau_{B^{0}} & =1.530 p s, & \tau_{B^{-}} & =1.65 p s, \\
M_{B_{s}} & =5.37 \mathrm{GeV}, & \tau_{B_{s}} & =1.47 \mathrm{ps},
\end{array}
$$

## References

[1] J.H. Christenson, J.W. Cronin, V.L. Fitch and R. Turlay, Evidence for the $2 \pi$ decay of the $k_{2}^{0}$ meson, Phys. Rev. Lett. 13 (1964) 138 [SPIRES].
[2] N. Cabibbo, Unitary symmetry and leptonic decays, Phys. Rev. Lett. 10 (1963) 531 [SPIRES].
[3] M. Kobayashi and T. Maskawa, CP violation in the renormalizable theory of weak interaction, Prog. Theor. Phys. 49 (1973) 652 [SPIRES].
[4] Particle Data Group collaboration, C. Amsler et al., Review of particle physics, Phys. Lett. B 667 (2008) $1 \mathrm{http}: / / \mathrm{pdg} .1 \mathrm{lbl} . g o v /[$ SPIRES].
[5] M.B. Gavela, P. Hernández, J. Orloff and O. Pene, Standard model CP-violation and baryon asymmetry, Mod. Phys. Lett. A 9 (1994) 795 [hep-ph/9312215] [SPIRES];
M.B. Gavela, M. Lozano, J. Orloff and O. Pene, Standard model CP violation and baryon asymmetry. Part 1: Zero temperature, Nucl. Phys. B 430 (1994) 345 [hep-ph/9406288] [SPIRES];
M.B. Gavela, P. Hernandez, J. Orloff, O. Pene and C. Quimbay, Standard model CP violation and baryon asymmetry. Part 2: Finite temperature, Nucl. Phys. B 430 (1994) 382 [hep-ph/9406289] [SPIRES];
P. Huet and E. Sather, Electroweak baryogenesis and standard model CP-violation, Phys. Rev. D 51 (1995) 379 [hep-ph/9404302] [SPIRES];
G.R. Farrar and M.E. Shaposhnikov, Baryon asymmetry of the universe in the standard electroweak theory, Phys. Rev. D 50 (1994) 774 [hep-ph/9305275] [SPIRES].
[6] UTfit collaboration, M. Bona et al., First evidence of new physics in $b \longleftrightarrow s$ transitions, arXiv:0803. 0659 [SPIRES];
M. Bona et al., New physics from flavour, arXiv:0906.0953 [SPIRES].
[7] A. Lenz and U. Nierste, Theoretical update of $B_{s}-\bar{B}_{s}$ mixing, JHEP 06 (2007) 072 [hep-ph/0612167] [SPIRES].
[8] CDF collaboration, T. Aaltonen et al., First flavor-tagged determination of bounds on mixing-induced CP-violation in $B_{s}^{0} \rightarrow J / \psi \phi$ decays, Phys. Rev. Lett. 100 (2008) 161802 [arXiv:0712.2397] [SPIRES].
[9] D0 collaboration, V.M. Abazov et al., Measurement of $B_{s}^{0}$ mixing parameters from the flavor-tagged decay $B_{s}^{0} \rightarrow J / \psi \phi$, Phys. Rev. Lett. 101 (2008) 241801 [arXiv:0802.2255] [SPIRES].
[10] C. Tarantino, B-Physics: theoretical predictions in the LHC Era, Nuovo Cim. 123B (2008) 437 [arXiv:0805.0698] [SPIRES].
[11] Heavy Flavor Averaging Group collaboration, E. Barberio et al., Averages of $b-h a d r o n$ and c-hadron properties at the end of 2007, http://www.slac.stanford.edu/xorg/hfag/ arXiv:0808. 1297 [SPIRES].
[12] B. Dutta and Y. Mimura, Large phase of $B_{s}-\bar{B}_{s}$ mixing in supersymmetric grand unified theories, Phys. Rev. D 78 (2008) 071702 [arXiv:0805.2988] [SPIRES];
M. Blanke, A.J. Buras, S. Recksiegel and C. Tarantino, The littlest higgs model with $T$-parity facing $C P$-violation in $B_{s}-\bar{B}_{s}$ mixing, arXiv:0805. 4393 [SPIRES];
A. Soni, A.K. Alok, A. Giri, R. Mohanta and S. Nandi, The fourth family: a natural explanation for the observed pattern of anomalies in $B-C P$ asymmetries, arXiv:0807.1971 [SPIRES];
M. Blanke, A.J. Buras, B. Duling, S. Gori and A. Weiler, $\Delta F=2$ observables and fine-tuning in a warped extra dimension with custodial protection, JHEP 03 (2009) 001 [arXiv:0809.1073] [SPIRES];
A. Datta and S. Khalil, Supersymmetry and CP-violation in $B_{s}^{0}-\bar{B}_{s}^{0}$ mixing and $B_{s}^{0} \rightarrow J / \psi \phi$ decay, Phys. Rev. D 80 (2009) 075006 [arXiv:0905.2105] [SPIRES].
[13] S. Chaudhuri, S.W. Chung, G. Hockney and J.D. Lykken, String consistency for unified model building, Nucl. Phys. B 456 (1995) 89 [hep-ph/9501361] [SPIRES].
[14] G. Cleaver, M. Cvetič, J.R. Espinosa, L.L. Everett and P. Langacker, Classification of flat directions in perturbative heterotic superstring vacua with anomalous U(1), Nucl. Phys. B 525 (1998) 3 [hep-th/9711178] [SPIRES];
G. Cleaver et al., Physics implications of flat directions in free fermionic superstring models. I: Mass spectrum and couplings, Phys. Rev. D 59 (1999) 055005 [hep-ph/9807479] [SPIRES].
[15] R. Blumenhagen, M. Cvetič, P. Langacker and G. Shiu, Toward realistic intersecting D-brane models, Ann. Rev. Nucl. Part. Sci. 55 (2005) 71 [hep-th/0502005] [SPIRES].
[16] G. Burdman, Constraints on the bulk standard model in the Randall-Sundrum scenario, Phys. Rev. D 66 (2002) 076003 [hep-ph/0205329] [SPIRES];
M. Blanke, A.J. Buras, B. Duling, S. Gori and A. Weiler, $\Delta F=2$ observables and fine-tuning in a warped extra dimension with custodial protection, JHEP 03 (2009) 001
[arXiv:0809.1073] [SPIRES];
M. Blanke, A.J. Buras, B. Duling, K. Gemmler and S. Gori, Rare K and B decays in a warped extra dimension with custodial protection, JHEP 03 (2009) 108 [arXiv:0812.3803] [SPIRES];
M.E. Albrecht, M. Blanke, A.J. Buras, B. Duling and K. Gemmler, Electroweak and flavour structure of a warped extra dimension with custodial protection, JHEP 09 (2009) 064 [arXiv:0903.2415] [SPIRES];
[17] P. Langacker, The physics of heavy $Z^{\prime}$ gauge bosons, arXiv:0801.1345 [SPIRES].
[18] P. Langacker and M. Plümacher, Flavor changing effects in theories with a heavy $Z^{\prime}$ boson with family nonuniversal couplings, Phys. Rev. D 62 (2000) 013006 [hep-ph/0001204] [SPIRES].
[19] V. Barger et al., Family non-universal $U(1)^{\prime}$ gauge symmetries and $b \rightarrow s$ transitions, Phys. Rev. D 80 (2009) 055008 [arXiv:0902.4507] [SPIRES].
[20] V. Barger, C.-W. Chiang, P. Langacker and H.-S. Lee, $Z^{\prime}$ mediated flavor changing neutral currents in B meson decays, Phys. Lett. B 580 (2004) 186 [hep-ph/0310073] [SPIRES].
[21] V. Barger, C.-W. Chiang, P. Langacker and H.-S. Lee, Solution to the $B \rightarrow \pi K$ puzzle in a flavor-changing $Z^{\prime}$ model, Phys. Lett. B 598 (2004) 218 [hep-ph/0406126] [SPIRES];
V. Barger, C.-W. Chiang, J. Jiang and P. Langacker, $B_{s}-\bar{B}_{s}$ mixing in $Z^{\prime}$ models with flavor-changing neutral currents, Phys. Lett. B 596 (2004) 229 [hep-ph/0405108] [SPIRES]; S. Baek, J.H. Jeon and C.S. Kim, $B_{s}^{0}-\bar{B}_{s}^{0}$ mixing and $B \rightarrow \pi K$ decays in stringy leptophobic $Z^{\prime}$, Phys. Lett. B 664 (2008) 84 [arXiv:0803.0062] [SPIRES];
X.-G. He and G. Valencia, $\bar{B}_{s} B_{s}$ mixing constraints on $F C N C$ and a non-universal $Z$ ', Phys. Rev. D 74 (2006) 013011 [hep-ph/0605202] [SPIRES];
R. Mohanta and A.K. Giri, Explaining $B \rightarrow K \pi$ anomaly with non-universal $Z^{\prime}$ boson, Phys. Rev. D 79 (2009) 057902 [arXiv:0812.1842] [SPIRES];
Q. Chang, X.Q. Li and Y.D. Yang, Constraints on the nonuniversal $Z^{\prime}$ couplings from $B \rightarrow \pi K, \pi K^{*}$ and $\rho K$ decays, arXiv:0903.0275 [SPIRES].
[22] A.J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, $B \rightarrow \pi \pi$, new physics in $B \rightarrow \pi K$ and implications for rare K and B decays, Phys. Rev. Lett. 92 (2004) 101804 [hep-ph/0312259] [SPIRES]; Anatomy of prominent $B$ and $K$ decays and signatures of CP-violating new physics in the electroweak penguin sector, Nucl. Phys. B 697 (2004) 133 [hep-ph/0402112] [SPIRES].
[23] G. Buchalla, A.J. Buras and M.E. Lautenbacher, Weak decays beyond leading logarithms, Rev. Mod. Phys. 68 (1996) 1125 [hep-ph/9512380] [SPIRES].
[24] R. Fleischer, S. Jager, D. Pirjol and J. Zupan, Benchmarks for the new-physics search through CP-violation in $B^{0} \rightarrow \pi^{0} K_{S}$, Phys. Rev. D 78 (2008) 111501 [arXiv:0806.2900] [SPIRES].
[25] M. Gronau and J.L. Rosner, Implications for CP asymmetries of improved data on $B \rightarrow K^{0} \pi^{0}$, Phys. Lett. B 666 (2008) 467 [arXiv:0807.3080] [SPIRES].
[26] C.-W. Chiang, M. Gronau, J.L. Rosner and D.A. Suprun, Charmless $B \rightarrow P$ decays using flavor $\mathrm{SU}(3)$ symmetry, Phys. Rev. D 70 (2004) 034020 [hep-ph/0404073] [SPIRES].
[27] S. Baek, C.-W. Chiang and D. London, The $B \rightarrow \pi K$ puzzle: 2009 update, Phys. Lett. B 675 (2009) 59 [arXiv:0903.3086] [SPIRES].
[28] M. Ciuchini, E. Franco, G. Martinelli, M. Pierini and L. Silvestrini, Searching for new physics with $B \rightarrow K \pi$ decays, Phys. Lett. B 674 (2009) 197 [arXiv:0811.0341] [SPIRES].
[29] A. Ali, G. Kramer and C.-D. Lu, Experimental tests of factorization in charmless non-leptonic two-body B decays, Phys. Rev. D 58 (1998) 094009 [hep-ph/9804363] [SPIRES].
[30] U. Nierste, $B_{d}$ and $B_{s}$ mixing: mass and width differences and CP-violation, Nucl. Phys. Proc. Suppl. 170 (2007) 135 [hep-ph/0612310] [SPIRES].
[31] Heavy Flavor Averaging Group (HFAG) collaboration, E. Barberio et al., Averages of b-hadron properties at the end of 2006, arXiv:0704.3575 [SPIRES].
[32] M. Wirbel, B. Stech and M. Bauer, Exclusive semileptonic decays of heavy mesons, Z. Phys. C 29 (1985) 637 [SPIRES];
M. Bauer, B. Stech and M. Wirbel, Exclusive nonleptonic decays of $D, D_{s}$ and $B$ mesons, $Z$. Phys. C 34 (1987) 103 [SPIRES];
J.G. Korner and G.A. Schuler, Exclusive semileptonic decays of bottom mesons in the spectator quark model, Z. Phys. C 38 (1988) 511 [Erratum ibid. C 41 (1988) 690] [SPIRES]; M. Bauer and M. Wirbel, Form-factor effects in exclusive d and B decays, Z. Phys. C 42 (1989) 671 [SPIRES].
[33] http://ckmfitter.in2p3.fr/plots_Summer08/ckmEval_results.html.
[34] M.S. Carena, A. Daleo, B.A. Dobrescu and T.M.P. Tait, $Z^{\prime}$ gauge bosons at the Tevatron, Phys. Rev. D 70 (2004) 093009 [hep-ph/0408098] [SPIRES].
[35] BABAR collaboration, B. Aubert et al., Direct CP, lepton flavor and isospin asymmetries in the decays $B \rightarrow K^{(*)} \ell^{+} \ell^{-}$, Phys. Rev. Lett. 102 (2009) 091803 [arXiv:0807.4119] [SPIRES]; Babar collaboration, K.T. Flood, Angular and isospin asymmetries in the decays $B \rightarrow K^{(*)} \ell^{+} \ell^{-}$, arXiv:0810.0837 [SPIRES].
[36] K. Cheung, C.-W. Chiang, N.G. Deshpande and J. Jiang, Constraints on flavor-changing Z' models by $B / s$ mixing, Z' production and $B / s \rightarrow \mu^{+} \mu^{-}$, Phys. Lett. B 652 (2007) 285 [hep-ph/0604223] [SPIRES].
[37] G. Buchalla and A.J. Buras, QCD corrections to rare $K$ and $B$ decays for arbitrary top quark mass, Nucl. Phys. B 400 (1993) 225 [SPIRES].
[38] CDF collaboration, R. Bernhard et al., A combination of CDF and D0 limits on the branching ratio of $B_{s}^{0}(d) \rightarrow \mu^{+} \mu^{-}$decays, hep-ex/0508058 [SPIRES].
[39] G. Buchalla, G. Hiller and G. Isidori, Phenomenology of nonstandard $Z$ couplings in exclusive semileptonic $b \rightarrow s$ transitions, Phys. Rev. D 63 (2000) 014015 [hep-ph/0006136] [SPIRES].


[^0]:    ${ }^{1}$ For family non-universal $\mathrm{U}(1)^{\prime}$ gauge symmetries in Randall-Sundrum scenarios, this is true in the limit that the mixing between the zero and KK modes is small.

[^1]:    ${ }^{2}$ Kinetic mixing allows the redefined $Z^{\prime}$ charges to have a component of weak hypercharge, which would otherwise not be allowed. This feature is irrelevant for the purposes of this paper.

[^2]:    ${ }^{3}$ These operators are not all independent. For couplings of four fermions of the same type, $\psi=\chi$, e.g. four charged leptons, one has $S_{m n}^{i j}=S_{i j}^{m n}, \tilde{S}_{m n}^{i j}=\tilde{S}_{i j}^{m n}$ and $T_{m n}^{i j}=\tilde{T}_{i j}^{m n}$.

[^3]:    ${ }^{4}$ Though the solutions to eq. (2.24) are not unique in the case with $n_{q}=1$, the physics is unaffected

[^4]:    ${ }^{5}$ This effect can also be seen by requiring a smaller C.L. for the fit of $\mathcal{C}_{\left(\phi, \eta^{\prime}, \rho, \omega, f_{0}\right) K_{S}}$ and $\mathcal{S}_{\left(\phi, \eta^{\prime}, \rho, \omega, f_{0}\right) K_{S}}$, which has been shown in the LR limit in figure 4 of [19].

